§ 1 Introduction

Today you are going to explore the physics of charges being pushed through an imperfectly conductive material.

We quantify a flow or charge in two ways. First we count the total charge per time passing, this is called the current.

$$I = \frac{dQ}{dt}$$

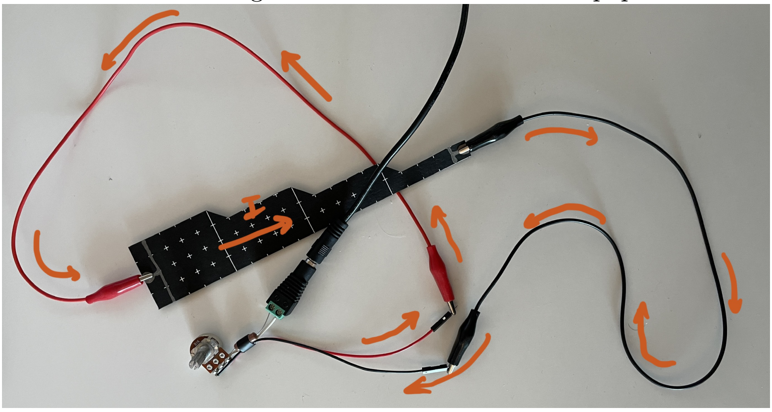
We can also refer to the intensity of the flow, that is the current per area, this is called the current density.

$$J = \frac{I}{A}$$

where A is the cross sectional area of the channel through which the charge is flowing.

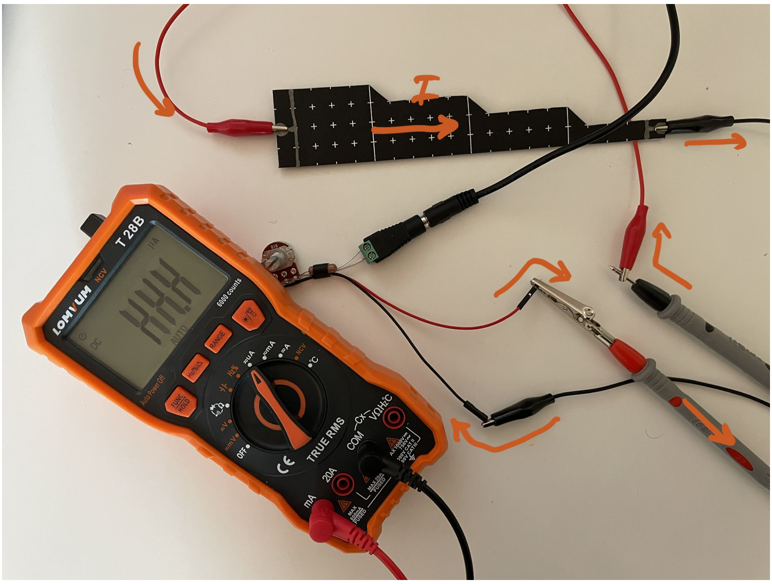
§ 2 Measuring Current

Connect the stepped black paper to the power supply as shown. Adjust the voltage of the power supply to 10 volts. This will create a flow or charges going through the paper as shown below. Also shown is the path of the current in the wires as it goes to and returns from the paper.



The DMM can measure the amount of the current, if you force the current to pass through the meter as shown below. Notice the connection of the leads to the DMM and the position of the dial are not the same as when you measure voltage. Write down the current you measure, be sure to note the units.

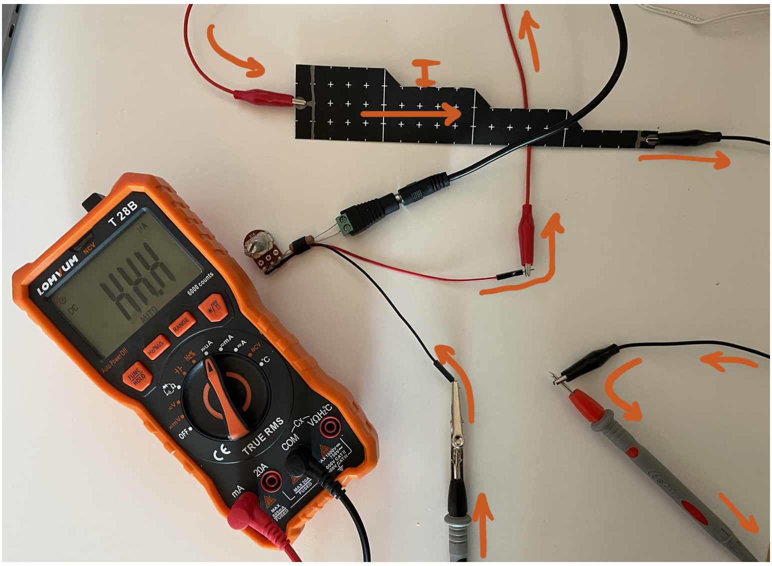
▷ QUESTION 1
Have you just measured the current before or after the current passed though the paper? Label your measured current as I_{before} or I_{after} as appropriate.



Now connect the DMM to the circuit as shown below, and measure the current again. Write down the value of the current.

▷ QUESTION 2
Have you just measured the current before or after the current passed though the paper? Label your measured current as I_{before} or I_{after} as appropriate.

▷ QUESTION 3
Compared I_{before} and I_{after}. Does the paper "use up" the current?

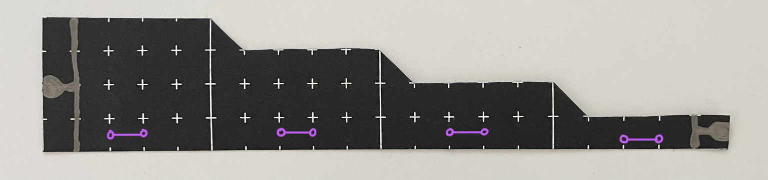


§ 3 Current Density and Electric Field Strength

Reconnect the stepped paper to the power supply as before without the DMM. Then measure the electric potential difference across the four pairs of points indicated in the image below. Us these four pairs of voltage measurements to estimate the electric field strength $E = \frac{\Delta V}{\Delta x}$ at those four locations. Also determine the current density at the four locations. The thickness of the paper is $\tau = 0.03$cm, and the widths are w =4cm, 3cm, 2cm and 1cm, so that the current densites can be computed via

$$J = \frac{I}{A} = \frac{I}{\tau w}$$

where I is the current you already measured.



▷ QUESTION 4

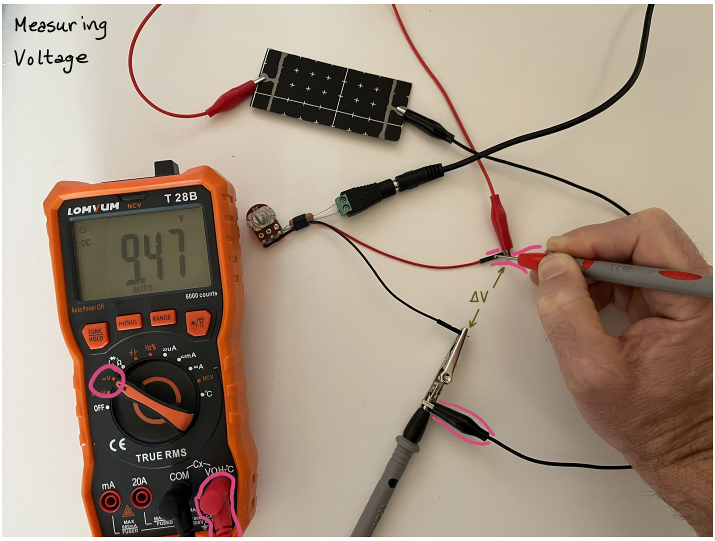
Plot J versus E. Does the current density increase with electric field? Is the increase linear? If so what is the slope?

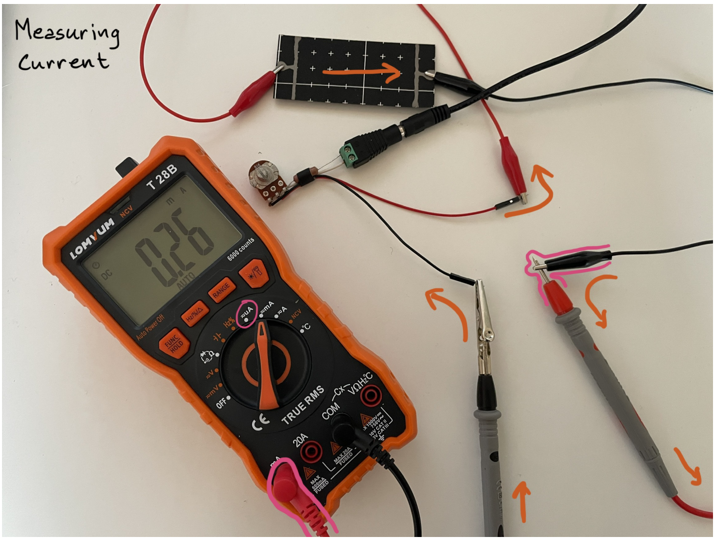
▷ QUESTION 5

Is the electric field strength bigger or smaller in the narrow section of the paper? Does this make sense given that E is the force per charge? You might thing about where the current is most "crowded"? Discuss this with your classmates and instructor. This is a point where there is a good opportunity to understand this phenomena in an intuitive way. Think again about the current density and where the current density is greatest, does this current density mirror describe "crowdedness" in a concrete way?

§ 4 Ohm's Law

We can also change the electric field strength and current density by changing the voltage applied to the paper. Connect the 8cm by 4cm piece of paper as shown below. This will allow you to measure the voltage applied to the paper. While measuring the voltage adjust the power supply to 11 volts. Now without changing the setting of the power supply change to measuring the current. You do this by changing the wiring connections as shown in the second image below. Record both the voltage and current. Repeat for measuring both the voltage and current for the voltage values of $V = 7\text{V}$, 5V, 3V, and 1V.





Now compute both the electric field strength $E = \frac{V}{\ell}$ and $J = \frac{I}{wt}$ were $\ell = 8\text{cm}$ is the length of the paper, $w = 4\text{cm}$ is the width of the paper and $t = 0.03\text{cm}$ is the thickness of the paper. Plot the current density versus the electric field strength.

▷ QUESTION 6
Based on your graph would it be accurate to say that $J = \sigma E$ for some constant σ? If so what is the value of σ?

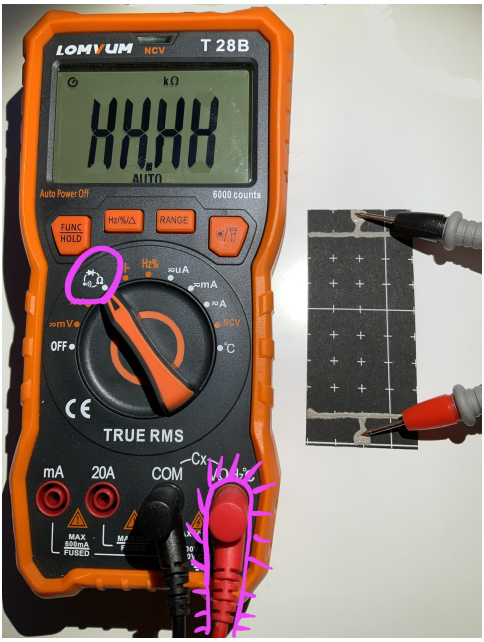
▷ QUESTION 7
Also graph V versus I. Would it be accurate to say that $V = RI$? If so what is the value of R.

▷ QUESTION 8
Materials that have the relationship $J = \sigma E$ and $V = RI$ are said to be ohmic, and these equations are both referred to as Ohm's Law. Is the paper ohmic?

§ 5 Conductivity and Resistance

The constant σ is called the conductivity and the constant R is called the resistance. The conductivity is a property of the material and the resistance is a function of both the conductivity and the geometry of the device.

The DMM can measure the resistance directly. Connect the DMM to the 8cm x 4cm piece of paper as shown in the image to the right to measure the resistance.



▷ Question 9
Compare the value of resistance you measure with the DMM and the value you got for R as the slope of the V versus I graph. Does the DMM give the same value?

Now measure the resistance of the three other rectangular pieces of paper and record their length ℓ, width w and resistance R. Note that ℓ is the distance between the metal electrodes, this distance needs be measured.

For this rectangular configuration we can write $J = \sigma E$ as

$$J = \sigma E$$
$$\longrightarrow \quad \frac{I}{wt} = \sigma \frac{V}{\ell}$$
$$\longrightarrow \quad \frac{\ell}{wt\sigma} I = V$$
$$\longrightarrow \quad V = \frac{\ell}{wt\sigma} I$$
$$\longrightarrow \quad R = \frac{\ell}{wt\sigma}$$

So we see that we expect the resistance to depend on the length the width the thickness and the conductivity.

To check to see if the above formula for the resistance is correct graph your measured value of R versus $\frac{\ell}{wt}$ for the four rectangular pieces of paper. $\frac{\ell}{wt}$ will be approximately $\frac{18\text{cm}}{4\text{cm}\ 0.03\text{cm}}$, $\frac{8\text{cm}}{4\text{cm}\ 0.03\text{cm}}$, $\frac{4\text{cm}}{4\text{cm}\ 0.03\text{cm}}$, $\frac{8\text{cm}}{2\text{cm}\ 0.03\text{cm}}$ but use your measured length ℓ.

▷ Question 10
Does your measured resistance follow the prediction $R = \frac{\ell}{wt\sigma}$? If so what is the value of the slope? How does this slope value reconcile with your previous measurement of the conductivity?