§ 1 Introduction

In this laboratory you will observe the effects of heat capacity, and the conduction of heat through the walls of a container, and check to see if the rate of heat flow is proportional to the temperature difference from one side fo the wall to the other.

§ 2 Background

The rate at which heat flows from region a to region b through a surface of thickness Δx and surface area A is

$$P = \frac{dQ_b}{dt} = kA\frac{dT}{dx} = kA\frac{T_a - T_b}{\Delta x}$$

where T_a is the temperature of region a and T_b is the temperature of region b, and k is the thermal conductivity of the material composing the surface. Notice that if $T_b > T_a$ that this rate is negative, that is region b is losing heat. This makes sense since heat moves from the hot to the cold side of the surface.

In our case we will have water in a cup at temperature T so we can write the rate at which heat flows into the water is

$$P = \frac{dQ}{dt} = kA\frac{T_0 - T}{\Delta x}$$

where T_0 is the temperature outside the cup. You will put hot water in the cup so this rate will be negative. Notice that this equation implies that the rate of heat flow P is a linear function of the temperature T and so if P was graphed versus T it would be a straight line.

We will set up the system so that the water only exchanges heat through the walls of the cup so we know that heat flowing in or out will change the internal energy of the water $dU = dQ$ and thus the temperature of the water changes as the heat flows $dQ = mc\, dT$, where m is the mass of the water and $c = 4.191\frac{\mathrm{J}}{\mathrm{g \cdot C^\circ}}$ is the specific heat of water.

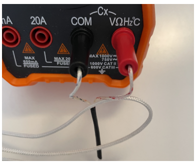
$$dQ = mc\, dT \quad \longrightarrow \quad P = \frac{dQ}{dt} = mc\frac{dT}{dt}$$

Thus we have a direct way to determine the rate at which heat is leaving the cup of water: $P = mc\frac{dT}{dt}$, in words that is compute the slope of the temperature versus time graph and then multiply that slope by mc. We will use this method to determine the rate of heat flow and then use this to see if the heat conduction equation $P = kA\frac{T_0 - T}{\Delta x}$ adequately describes the cooling of a cup water.

§ 3 Measuring the temperature of hot water as it cools.

Procedure 3.a

1) First find the following items in your lab kit.
   - digital multimeter (DMM), and the temperature probe in its box.
   - stopwatch
   - plastic cup
   - round styrofoam block
   - square styrofoam block
   - camera tripod (from last semester)
   - weight scale (from last semester)
2) Plug the temperature probe into the DMM.

   The black wire of the temperature probe plugs into the connector labeled COM and the red wire plugs into the connector labeled VΩHz°C. Next rotate the dial of the DMM to the °C setting. The DMM will now read the temperature of the probe.
3) Now set the stopwatch next to the DMM and place the camera on the tripod so that it can see both the screen of the DMM and the screen

of the stopwatch. Start the stopwatch and make a short video of the two screens, hold the tip of the probe between your fingers to warm it up during the video, so you can see the change in the temperature reading in the video. Look at the video to be sure that you can see both the time and the temperature readings in the video. Delete the video.

4) Place square styrofoam block with the aluminum side up on the table next to the DMM. Fill the plastic cup with water to about 5mm from the top. Place the plastic cup on top of the block. Place the round styrofoam block (the lid) into the mouth of the cup with the aluminum side down. It should be a snug fit but not tight and the block should only go in a few millimeters. If it is tight you can squash the styrofoam a little to make it smaller. Now place the temperature probe through the hole in the round styrofoam and into the water so that the tip is in the center of the volume of water, then bend the wire over so that it stays at this location. Make sure everything is stable.
5) Boil some water. When the water is boiling do the following in order.

   a) Dump out the water that is in the cup and put the cup back on the styrofoam block.
   b) Start the stop watch.
   c) Start the video recording of the two screens.
   d) Pour the hot water into the cup.
   e) Replace the styrofoam lid and temperature probe.
   f) Wait one hour, but stick around because the DMM will try to go to sleep every ten minutes. You can prevent the sleep by pushing the orange button with the flashlight symbol on it. Press it twice so that the light does not stay on. You need to keep doing this every ten minutes.
   g) Stop the video recording when you have recorded an hour of the water coolling.
   h) Remove the lid.
   i) Weigh the cup with the water in it. Weigh the cup without the water in it, and record the difference (the weight of the water).

   The above steps are all simple but can go wrong. While I was running trials of this lab, my camera ran out of battery, the camera ran out of memory, the DMM went to sleep . . . etc. Having to start over after 55 minutes is demoralizing. So set things up carefully. Plug the camera/phone into a power source. Set the camera so that it records low resolution video at a slow frame rate so that you don't run out of memory. If the camera has a time-laps mode you can use that.

§ 4 Data analysis

Now look at the video you recorded, finding the frames where the temperature reading changes. Record in a plain text file for each transition the temperature and the time. For example suppose that the transition is from 59°C to 58°C and the stopwatch reads  when the transition occurs. You would add one line to the text file like this

58, 12, 27

always ignoring the hundredths of seconds shown on the stopwatch. If you go over one hour then you will have times like 1:07:32 which should be recorded as

xx, 67, 32

with xx the temperature. Save the text file as `temp.csv` .

Get the application for this lab [macOS, MSwin] from the class website. Run `Lab1p1`, there will be a box to fill in the mass of the water in grams and a button to select the data file `temp.csv`. Once these are entered two graphs will be produced. The graph on the left is the temperature versus time, and a best fit curve. It is possible that at the beginning the water was still reaching equilibrium with the cup, this would be evident in that the data would not fit the curve at the beginning. You can exclude this early data by setting the start time to a time after it is settled in. The program will ignore the data before this time.

▷ QUESTION 1
How well does the theoretical curve $T(t) = T_\infty + (T_0 - T_\infty)e^{-t/\tau}$ fit the experimental data?

▷ QUESTION 2
Now look at the graph on the right. Here is plotted the slope of the graph on the left (multiplied by mc) versus the temperature, in other words the rate of heat flow $P = mc\frac{dT}{dt}$ versus the temperature T. The right graph is computed from the left graph, but the connection is not so obvious. Spend a moment now making sure you understand what is graphed.

Do this by computing the rate of heat flow at two different temperatures one large and one small (say 75°C and 40°C). To estimate the slope at 75°C look at your data table and find the time t_{76} for 76°C and the time t_{74} for 74°C. Now you can compute the rate of heat flow when the temperature is 75°C as

$$P_{75} = mc\frac{dT}{dt} \approx mc\frac{T_f - T_i}{t_f - t_i} = mc\frac{74^\circ\mathrm{C} - 76^\circ\mathrm{C}}{t_{74} - t_{76}}$$

The value of mc is printed on the second graph. Plot this point $(75^\circ\mathrm{C}, P_{75})$ on the second graph. Is it similar to the point the computer graphed for the temperature 75°C?

Repeat the above at 40°C.

A best fit straight line has also been graphed in second graph. Does the straight line do a good job of fitting your measured values of P versus T?

▷ QUESTION 3
Now consider the prediction that the rate of heat transfer should be proportional to the temperature difference:

$$P = kA\frac{dT}{dx} = kA\frac{T_0 - T}{\Delta x} = \frac{kA}{\Delta x}(T_0 - T).$$

If this prediction were true what would you expect the graph of P versus T to look like? Does your measured data look like that? Could you use the slope of your best fit line to determine $\frac{kA}{\Delta x}$?