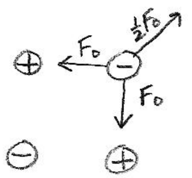
1.1 The magnitude of the force between all pairs that are a distance a apart is $F_0 = \frac{q^2}{4\pi\epsilon_0 a^2}$. Now notice that since the diagonal is $\sqrt{2}a$ that the magnitude of the force for pairs across the diagonal will be $\frac{q^2}{4\pi\epsilon_0(\sqrt{2}a)^2} = \frac{1}{2}F_0$.

Now lets draw the forces on the upper right charge



We see that the leftward and downward force will add together to make a force that is along the diagonal toward the center and with a magnitude of $\sqrt{2}F_0$. The $\frac{1}{2}F_0$ force is away from the center, so the net force is toward the center and of magnitude $F = (\sqrt{2} - \frac{1}{2})F_0$. By drawing the forces on the other three charges you can quickly see that all charges feel a force toward the center and of equal magnitude.

1.2 Let

$$\vec{r}_1 = (0.2\text{m})\hat{\jmath} \quad \text{and} \quad q_1 = 1.0\mu\text{C}$$
$$\vec{r}_2 = (-0.3\text{m})\hat{\jmath} \quad \text{and} \quad q_2 = -2.0\mu\text{C}$$
$$\vec{r}_3 = (0.4\text{m})\hat{\imath} \quad \text{and} \quad q_3 = 3.0\mu\text{C}$$

In preparation we compute the following quantities

$$\vec{r}_1 - \vec{r}_2 = (0.5\text{m})\hat{\jmath} \quad \text{and} \quad |\vec{r}_1 - \vec{r}_2| = 0.500\text{m}$$
$$\vec{r}_2 - \vec{r}_3 = -(0.4\text{m})\hat{\imath} - (0.3\text{m})\hat{\jmath} \quad \text{and} \quad |\vec{r}_2 - \vec{r}_3| = 0.500\text{m}$$
$$\vec{r}_3 - \vec{r}_1 = (0.4\text{m})\hat{\imath} - (0.2\text{m})\hat{\jmath} \quad \text{and} \quad |\vec{r}_3 - \vec{r}_1| = 0.447\text{m}$$

and

$$q_1q_2/4\pi\epsilon_0 = -0.018\text{N}\cdot\text{m}^2$$
$$q_2q_3/4\pi\epsilon_0 = -0.054\text{N}\cdot\text{m}^2$$
$$q_3q_1/4\pi\epsilon_0 = 0.027\text{N}\cdot\text{m}^2$$

$$\begin{aligned}
\vec{F}_1 &= \vec{F}_{12} + \vec{F}_{13} \\
&= \frac{q_1q_2}{4\pi\epsilon_0}\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3} + \frac{q_1q_3}{4\pi\epsilon_0}\frac{\vec{r}_1 - \vec{r}_3}{|\vec{r}_1 - \vec{r}_3|^3} \\
&= \frac{q_1q_2}{4\pi\epsilon_0}\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3} - \frac{q_3q_1}{4\pi\epsilon_0}\frac{\vec{r}_3 - \vec{r}_1}{|\vec{r}_3 - \vec{r}_1|^3} \\
&= -0.018\text{N}\cdot\text{m}^2\frac{(0.5\text{m})\hat{\jmath}}{(0.500\text{m})^3} - 0.027\text{N}\cdot\text{m}^2\frac{(0.4\text{m})\hat{\imath} - (0.2\text{m})\hat{\jmath}}{(0.447\text{m})^3} \\
&= (-0.121\text{N})\hat{\imath} + (-0.012\text{N})\hat{\jmath}
\end{aligned}$$

Similarly

$$\begin{aligned}
\vec{F}_2 &= \vec{F}_{21} + \vec{F}_{23} \\
&= 0.018\text{N}\cdot\text{m}^2\frac{(0.5\text{m})\hat{\jmath}}{(0.500\text{m})^3} - 0.054\text{N}\cdot\text{m}^2\frac{-(0.4\text{m})\hat{\imath} - (0.3\text{m})\hat{\jmath}}{(0.500\text{m})^3} \\
&= (0.173\text{N})\hat{\imath} + (0.202\text{N})\hat{\jmath}
\end{aligned}$$

and

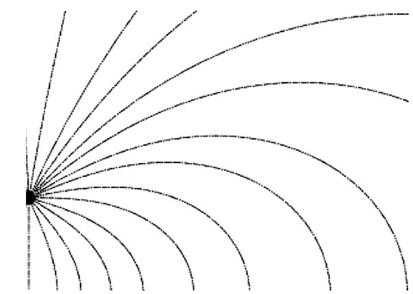
$$\begin{aligned}
\vec{F}_3 &= \vec{F}_{31} + \vec{F}_{32} \\
&= 0.027\text{N}\cdot\text{m}^2\frac{(0.4\text{m})\hat{\imath} - (0.2\text{m})\hat{\jmath}}{(0.447\text{m})^3} + 0.054\text{N}\cdot\text{m}^2\frac{-(0.4\text{m})\hat{\imath} - (0.3\text{m})\hat{\jmath}}{(0.500\text{m})^3} \\
&= (-0.052\text{N})\hat{\imath} + (-0.190\text{N})\hat{\jmath}
\end{aligned}$$

1.3

(a)

$$\begin{aligned}
\vec{E}(\vec{r}) = \vec{E}_1 + \vec{E}_2 &= \frac{q_1}{4\pi\epsilon_0}\frac{\vec{r} - \vec{r}_1}{|\vec{r} - \vec{r}_1|^3} + \frac{q_2}{4\pi\epsilon_0}\frac{\vec{r} - \vec{r}_2}{|\vec{r} - \vec{r}_2|^3} \\
\longrightarrow \quad \vec{E}(x\hat{\imath}) &= \frac{q}{4\pi\epsilon_0}\frac{x\hat{\imath} - a\hat{\jmath}}{|x\hat{\imath} - a\hat{\jmath}|^3} + \frac{-q}{4\pi\epsilon_0}\frac{x\hat{\imath} + a\hat{\jmath}}{|x\hat{\imath} + a\hat{\jmath}|^3} \\
&= \frac{q}{4\pi\epsilon_0}\frac{x\hat{\imath} - a\hat{\jmath}}{(x^2 + a^2)^{3/2}} + \frac{-q}{4\pi\epsilon_0}\frac{x\hat{\imath} + a\hat{\jmath}}{(x^2 + a^2)^{3/2}} \\
&= \frac{q}{4\pi\epsilon_0}\frac{-2a\hat{\jmath}}{(x^2 + a^2)^{3/2}}
\end{aligned}$$

(b)



1.4 We will use polar coordinates to describe the path of our line charge, $\vec{r}_s = R\cos\theta\hat{\imath} + R\sin\theta\hat{\jmath}$ and $dq = \frac{Q}{2\pi}d\theta$. The field point is $\vec{r} = z\hat{k}$, so that

$$\vec{r} - \vec{r}_s = -R\cos\theta\hat{\imath} - R\sin\theta\hat{\jmath} + z\hat{k}$$

and

$$|\vec{r} - \vec{r}_s| = \sqrt{R^2\cos^2\theta + R^2\sin^2\theta + z^2} = \sqrt{R^2 + z^2}$$

Fortunately this does not depend on the angle θ, so we will be able to take it out of the integral that is to follow.

$$\begin{aligned}\vec{E}(\vec{r}) &= \int \frac{dq}{4\pi\epsilon_0}\frac{\vec{r}-\vec{r}_s}{|\vec{r}-\vec{r}_s|^3} \\ &= \int_0^{2\pi} \frac{\frac{Q}{2\pi}d\theta}{4\pi\epsilon_0}\frac{-R\cos\theta\hat{\imath}-R\sin\theta\hat{\jmath}+z\hat{k}}{(R^2+z^2)^{3/2}} \\ &= \frac{1}{4\pi\epsilon_0}\frac{Q}{2\pi}\frac{1}{(R^2+z^2)^{3/2}}\int_0^{2\pi} d\theta(-R\cos\theta\hat{\imath}-R\sin\theta\hat{\jmath}+z\hat{k}) \\ &= \frac{1}{4\pi\epsilon_0}\frac{Q}{2\pi}\frac{1}{(R^2+z^2)^{3/2}}(0\hat{\imath}-0\hat{\jmath}+2\pi z\hat{k}) \\ &= \frac{Q}{4\pi\epsilon_0}\frac{z\hat{k}}{(R^2+z^2)^{3/2}}\end{aligned}$$

1.5 $\Phi = \frac{q_{\rm in}}{\epsilon_o}$ so $\Phi_1 = \frac{-Q}{\epsilon_o}$, $\Phi_2 = 0$, $\Phi_3 = \frac{-2Q}{\epsilon_o}$, $\Phi_4 = 0$.

1.6 There is no flux through the faces touching the charge because the electric field is perpendicular to the normal to these surfaces, that is, the field is parallel to the surface. By symmetry the other three faces all must have the same flux, call it ϕ. If we take eight such cubes and put them together in a super-cube that is twice as big, we can put the charge at the center of this super-cube. Each of the cubes will have the charge at a corner, so each will have the flux we are trying to find on the outside surfaces, there are 24 such surfaces surrounding the charge so the net flux through the super-cube is $\phi_{\rm net} = 24\phi$. But by Gauss's law the net flux is $Q_{\rm in}/\epsilon 0$ so

$$24\phi = \frac{Q_{\rm in}}{\epsilon 0} \longrightarrow \phi = \frac{Q_{\rm in}}{24\epsilon 0}$$

1.7 Use a gaussian surface that is a sphere of radius r, centered on the charge. Since the field is radial, it is everywhere normal to the surface. Also the field is uniform over this surface since there is no preferred direction. Thus

$$\oint \vec{E}\cdot d\vec{A} = EA = E\ 4\pi r^2$$

Putting this into Gauss's law and noting that $Q_{\rm in}$ is the charge of the particle, we find that

$$E\ 4\pi r^2 = \frac{q}{\epsilon_0} \longrightarrow E = \frac{q}{4\pi\epsilon_0}\frac{1}{r^2}$$

1.8 Since the charge distribution is sphericaly symetric the field will radiate straight out from the center of the charge distribution. Consider a sphere or radius r with its center at the center of the charge distribution. The field will at all points on this sphere be perpendicular to the surface of the sphere and of constant magnitude. Thus the flux will be simply the magnitude of the E-field times the area of the sphere $\phi = EA = E4\pi r^2$. But also the flux will be given by Gauss's law to be $\phi = q_{\rm in}/\epsilon_o$ Equating these two expressions for the flux and solving for the field we find

$$E = \frac{q_{\rm in}}{\epsilon_o 4\pi r^2}.$$

If we are outside the sphere $q_{\rm in} = Q$. While if $r < R$ then $q_{\rm in} = \rho V = \frac{Q}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi r^3 = Q\frac{r^3}{R^3}$. Thus we find

$$E = \begin{cases} \frac{Q}{\epsilon_o 4\pi r^2} & \text{if } r > R \\ \frac{Qr}{\epsilon_o 4\pi R^3} & \text{if } r < R \end{cases}$$

1.9 As in the previous problems, the electric field at any distance r from the sphere's center will be given by

$$E = \frac{q_{enc}}{\epsilon_o 4\pi r^2},$$

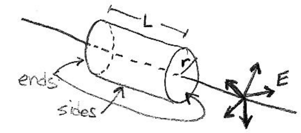
where q_{enc} is the charge enclosed by a spherical Gaussian surface of radius r. So let's find the charge inside a radius r that is less then a.

$$q_{enc} = \int_V \rho\ dV = \int_0^r \rho\ 4\pi r^2 dr = \int_0^r \rho_o\left(\frac{r}{a}\right)^2 4\pi r^2 dr = \frac{4\pi\rho_o r^5}{5a^2}$$

Thus

$$E = \begin{cases} \frac{\rho_o a^3}{5\epsilon_o r^2} & \text{if } r > a \\ \frac{\rho_o r^3}{5\epsilon_o a^2} & \text{if } r < a \end{cases}$$

1.10 Let our Gaussian surface be a cylinder of length L and radius r with the line charge on the axis of the cylinder.



The flux is zero through the ends of the cylinder because the field is parallel to the plane of the ends. While the field is parallel to the normal to the surface at all points on the sides of the cylinder, so that $\vec{E}\cdot d\vec{A} = E\ dA$. Also the field will be uniform in strength at all points on the sides of the surface because the sides are all at a distance r from the line charge. Thus

$$\begin{aligned}\oint \vec{E}\cdot d\vec{A} &= \int_{\rm ends}\vec{E}\cdot d\vec{A} + \int_{\rm sides}\vec{E}\cdot d\vec{A} \\ &= 0 + \int_{\rm sides} E dA = E\int_{\rm sides} dA = EL2\pi r\end{aligned}$$

We have compute the left side of Gauss's law. Now we need to compute the charge inside of this surface. Since the surface contains a length L of the line charge, the charge inside will be $Q_{\rm in} = \lambda L$. So now we can use Gauss's law to find the field strength.

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

$$\longrightarrow\ EL2\pi r = \lambda L\epsilon_0 \ \longrightarrow\ E = \frac{\lambda}{2\pi\epsilon_0 r}$$

1.11 Pick our Gaussian surface to be a cylinder of radius r and length L that is coaxial with the charge distribution. Then we know that the field is perpendicular to the sides and parallel to the ends and thus the flux is

$$\phi = EA_{\text{sides}} = E2\pi rL.$$

But by Gauss's Law we know that the flux is

$$\phi = \frac{q_{\text{in}}}{\epsilon_o} = \frac{\rho V_{\text{in}}}{\epsilon_o} = \frac{\rho\pi r^2 L}{\epsilon_o}$$

equating these two expressions for the flux and solving for the field we find $E = \rho r/2\epsilon_o$.

1.12 Once again, because the charge distribution is spherically symmetric, we find that

$$E = \frac{q_{\text{in}}}{4\pi\epsilon_0 r^2}.$$

Since all of the charge is on the surface of the conductor, if $r > R$ then $q_{\text{in}} = Q$, and if $r < R$ then $q_{\text{in}} = 0$. So outside the surface we find $E = \frac{Q}{4\pi\epsilon_0 r^2}$ and anywhere inside $E = 0$.

1.13 Let the two bottom charges be 1 and 2.

$$\vec{F} = \Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 = \frac{q_1 q}{4\pi\epsilon_0}\frac{\vec{r}-\vec{r}_1}{|\vec{r}-\vec{r}_1|^3} + \frac{q_2 q}{4\pi\epsilon_0}\frac{\vec{r}-\vec{r}_1}{|\vec{r}-\vec{r}_2|^3}$$

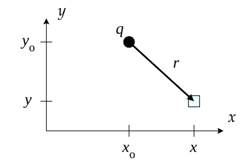
$$= \frac{(2\mu\text{C})(7\mu\text{C})}{4\pi\epsilon_0}\frac{(0.5\text{m})(\frac{1}{2}\hat{\imath}+\frac{\sqrt{3}}{2}\hat{\jmath})}{(0.5\text{m})^3} + \frac{(-4\mu\text{C})(7\mu\text{C})}{4\pi\epsilon_0}\frac{(0.5\text{m})(-\frac{1}{2}\hat{\imath}+\frac{\sqrt{3}}{2}\hat{\jmath})}{(0.5\text{m})^3}$$

$$= \frac{(2\mu\text{C})(7\mu\text{C})}{4\pi\epsilon_0}\frac{(\frac{1}{2}\hat{\imath}+\frac{\sqrt{3}}{2}\hat{\jmath})}{(0.5\text{m})^2} + \frac{(-4\mu\text{C})(7\mu\text{C})}{4\pi\epsilon_0}\frac{(-\frac{1}{2}\hat{\imath}+\frac{\sqrt{3}}{2}\hat{\jmath})}{(0.5\text{m})^2}$$

$$= (0.76\hat{\imath} + -0.44\hat{\jmath})\text{N}$$

1.14 Since we are mostly water we can compute the number of protons by computing the number of protons in about 75 kg of water. Water is 18 grams per mole and contains 10 protons per molecule. Thus there are $\frac{75}{0.018}N_A \approx 2.5\times10^{27}$ molecules and 2.5×10^{28} protons. If there were 1% more electrons than this there would be a net charge of $q = -2.5\times10^{26}e \approx 4\times10^{7}$C. Arms length is about $r = 1.0$m, so the force would be $F \approx k\frac{q^2}{r^2} = 1.4\times10^{25}$N. While the "weight" of the world is $m_E g = 6\times10^{25}$N. So we see that these are of the same order of magnitude.

1.15 From the figure



we see that

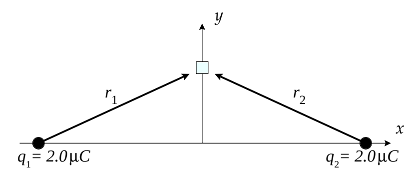
$$\vec{r} = (x-x_o)\hat{\imath} + (y-y_o)\hat{\jmath}$$

$$r = \sqrt{(x-x_o)^2 + (y-y_o)^2}$$

Thus we can write

$$\vec{E} = k\frac{q}{r^2}\hat{r} = k\frac{q}{r^2}\frac{\vec{r}}{r} = kq\frac{\vec{r}}{r^3} = kq\frac{(x-x_o)\hat{\imath}+(y-y_o)\hat{\jmath}}{[(x-x_o)^2+(y-y_o)^2]^{3/2}}$$

1.16 From the figure



$$\vec{r}_1 = (1.0\text{m})\hat{\imath} + (0.5\text{m})\hat{\jmath}$$

$$\vec{r}_2 = (-1.0\text{m})\hat{\imath} + (0.5\text{m})\hat{\jmath}$$

$$r = r_1 = r_2 = 1.1\text{m}$$

Thus we can write the E-field as

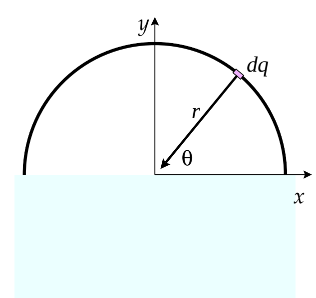
$$\vec{E} = \Sigma\vec{E}_i = \vec{E}_1 + \vec{E}_2 = kq_1\frac{\vec{r}_1}{r_1^3} + kq_2\frac{\vec{r}_2}{r_2^3} = \frac{kq}{r^3}(\vec{r}_1 + \vec{r}_2)$$

$$= \frac{k(2.0\mu\text{C})}{(1.1\text{m})^3}\left((0.0\text{m})\hat{\imath} + (1.0\text{m})\hat{\jmath}\right) = 1.35\times10^4\,\frac{\text{N}}{\text{C}}\,\hat{\jmath}$$

If a charge $q_o = -3.0\mu$C is at this point it will feel a force

$$\vec{F} = q_o\vec{E} = -4.06\times10^{-2}\text{N}\ \hat{\jmath}$$

1.17



$$D = 2\pi r \rightarrow r = \frac{14\text{cm}}{\pi} = 0.045\text{m}$$

$$\vec{r} = -r\cos\theta\hat{\imath} - r\sin\theta\hat{\jmath}$$

$$dq = \frac{-7.5\mu\text{C}}{\pi}d\theta = \lambda d\theta$$

$$\vec{E} = \int k\frac{dq}{r^2}\hat{r} = \frac{k\lambda}{r^3}\int_0^\pi \vec{r}d\theta = -\frac{k\lambda}{r^2}\int_0^\pi(\cos\theta\hat{\imath} + \sin\theta\hat{\jmath})d\theta$$
$$= -\frac{k\lambda}{r^2}\left[\sin\theta\hat{\imath} - \cos\theta\hat{\jmath}\right]_0^\pi = -2\frac{k\lambda}{r^2}\hat{\jmath} = -2.1\times10^7\frac{\text{N}}{\text{C}}\hat{\jmath}$$

1.18 The work done by the field goes into changing the kinetic energy of the electron so

$$\Delta K = W \longrightarrow \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = F\Delta x$$

Notice that the force and Δx are in opposite directions so that regardless of the choice of coordinates the product $F\Delta x$ will be negative. Now use $v_f = 0$ and $F = qE = -eE$ and solve the above equation for Δx

$$\Delta x = \frac{mv_i^2}{2eE} = 2.6\text{cm}$$

1.19 There is no force in the horizontal direction so the proton will have a constant speed of $4.50\times10^5\frac{\text{m}}{\text{s}}$ in the horizontal direction. Thus it will take a time of $\frac{0.05\text{m}}{4.50\times10^5\frac{\text{m}}{\text{s}}} = 1.11\times10^{-7}$s to travel 5 cm horizontally.

In the vertical direction there is a force $qE = eE$ so the acceleration is $a = F/m = eE/m$. We can now use the constant acceleration equation to find the displacement

$$\Delta x = v_o t + \tfrac{1}{2}at^2 = 0 + \frac{eE}{2m}(1.11\times10^{-7}\text{s})^2 = 5.67\text{mm}.$$

At this time the vertical velocity will be

$$v = v_0 + at = 0 + at = -1.02\times10^5\tfrac{\text{m}}{\text{s}}.$$

1.20 $\Sigma\vec{F} = m\vec{a} \longrightarrow \vec{F}_e + \vec{T} + \vec{F}_g = 0 \longrightarrow q\vec{E} + \vec{T} + \vec{F}_g = 0$

$$q(E_x\hat{\imath} + E_y\hat{\jmath}) + (-T\sin\theta\hat{\imath} + T\cos\theta\hat{\jmath}) - mg\hat{\jmath} = 0$$

The x and y components of this equation are

$$x: \quad qE_x - T\sin\theta + 0 = 0$$
$$y: \quad qE_y + T\cos\theta - mg = 0$$

Eliminating T and solving for q we find

$$q = \frac{mg}{E_x\cot\theta + E_y} = 1.09\times10^{-8}\text{C}$$

Putting this value back into the x equation we find $T = 5.4\times10^{-3}$N.

1.21 For the vertical side the normal and the field are in opposite directions so the dot product becomes

$$\Phi_a = \vec{E}\cdot\Delta\vec{A}_a = -E\Delta\vec{A}_a = -2.34\times10^2\frac{\text{Nm}^2}{\text{C}}.$$

For the slanted side

$$\Delta A_b = (0.30\text{m}\times\frac{0.10\text{m}}{\cos60°}) = \frac{\Delta A_a}{\cos60°}.$$

While the angle between the normal and the field is 60° so

$$\Phi_a = \vec{E}\cdot\Delta\vec{A}_b = E\Delta A_b\cos60° = E\frac{\Delta A_a}{\cos60°}\cos60° = -\Phi_a$$

The flux on the other three sides is zero since the field is parallel to the faces. Thus the net flux is zero, as it will be for any closed surface with no charge inside.

1.22 Since the net flux is zero the flux exiting the paraboloidal surface must be equal to the flux entering the flat side $\Phi = \pi r^2 E_o$.

1.23 The total flux is $\Phi = \frac{q_{\text{in}}}{\epsilon_o}$ regardless of where the charge is place in the box. If the charge is at the center then the flux will be equaly divided amongst the six faces and thus the flux through each face will be $\Phi_{\text{face}} = \frac{q_{\text{in}}}{6\epsilon_o}$.

1.24 Since the net flux is zero the flux exiting the hemispherical surface must be equal to the flux entering the flat side. Also we know that the hemispherical surface must catch half of what would go through a full sphere so $\Phi = \frac{1}{2}\frac{q}{\epsilon_o}$.

1.25 First note that $\vec{r} = -x\hat{\imath}$ so $r = x$ and $\hat{r} = -\hat{\imath}$. Thus

$$\vec{E} = \int k\frac{dq}{r^2}\hat{r} = \int_a^b k\frac{\lambda dx}{x^2}(-\hat{\imath}) = -\int_a^b k\frac{cx^n dx}{x^2}\ \hat{\imath}$$
$$= -\int_a^b kcx^{n-2}dx\ \ \hat{\imath} = -kc[x^{n-1}]_a^b\ \hat{\imath}$$

2.1 $\delta U = q\Delta V = (0.08\text{C})(9\text{V}) = 0.72\text{J}$.

2.2

$$\begin{aligned} \Delta K + \Delta U &= 0 \\ \longrightarrow\ K_f - K_i &= -\Delta U \\ K_f - 0 &= -q\Delta V \\ \tfrac{1}{2}mv_f^2 &= -(-e)(+1000\text{V}) \\ \longrightarrow\ v_f &= \sqrt{\frac{2e}{m}(1000\text{V})} = 1.87\times10^7\,\tfrac{\text{m}}{\text{s}} \end{aligned}$$

2.3 First we need to pick a path from the starting point to the ending point. A straight line will do. Let $\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} = (1.0\text{m})t\hat{\imath} + (2.0\text{m})t\hat{\jmath}$.

$$\vec{dr} = \frac{d\vec{r}}{dt}dt = \left(\frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}\right)dt = ((1.0\text{m})\hat{\imath} + (2.0\text{m})\hat{\jmath})dt$$

so that

$$\begin{aligned} \Delta V &= -\int_A^B \vec{E}\cdot\vec{dr} \\ &= -\int_0^1 \vec{E}(\vec{r}(t))\cdot\frac{d\vec{r}}{dt}dt \\ &= -\int_0^1 (ay\hat{\imath} + ax\hat{\jmath})\cdot((1.0\text{m})\hat{\imath} + (2.0\text{m})\hat{\jmath})dt \\ &= -\int_0^1 (a(2.0\text{m})t\hat{\imath} + a(1.0\text{m})t\hat{\jmath})\cdot((1.0\text{m})\hat{\imath} + (2.0\text{m})\hat{\jmath})dt \\ &= -\int_0^1 (4.0\text{m})at\ dt \\ &= -(2.0\text{m})a \end{aligned}$$

2.4

$$\begin{aligned} \vec{E} &= -\vec{\nabla}V \\ &= -\left[\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right] \\ &= -\frac{\partial}{\partial x}axyz\ \hat{\imath} - \frac{\partial}{\partial y}axyz\ \hat{\jmath} - \frac{\partial}{\partial z}axyz\ \hat{k} \\ &= ayz\ \hat{\imath} + axz\ \hat{\jmath} + axy\ \hat{k} \end{aligned}$$

2.6 Consider a closed surface that is totally within the outer conductor and surrounds the inside surface of the outer conductor, as indicated by the dotted line in the figure. Since there is no field inside a conductor, the electric flux through this surface is zero; $\phi_e = 0$. Thus by Gauss's law we know that the $Q_{\text{in}} = \epsilon_0\phi_e = 0$. But the charge inside is the charge on the inside surface of the outer conductor, Q_{inside}, plus the charge on the inner conductor, Q_a. Thus

$$0 = Q_{\text{in}} = Q_{\text{inside}} + Q_a \ \longrightarrow\ Q_{\text{inside}} = -Q_a.$$

Next we note that the total charge on the outer conductor is the sum of the charge on it's inside and outside surfaces so that

$$\begin{aligned} Q_b &= Q_{\text{inside}} + Q_{\text{outside}} \\ \longrightarrow\ Q_{\text{outside}} &= Q_b - Q_{\text{inside}} = Q_b - (-Q_a) = Q_b + Q_a \end{aligned}$$

2.7 Consider a Gaussian surface that is a sphere at a radius of 4.5 cm. Since this surface is totally within the body of the conductor we know that the flux is zero since the field is zero in a conductor. But this tells us that the charge inside the surface is zero. Thus we know that the inside surface of the shell must carry a charge equal and opposite to the charge of the point charge. Thus the inside surface carries a charge of -2.0μC. In order for the shell to have a net charge of 10μC then the charge on the outside surface must be 12μC.

$$\begin{aligned} \sigma_{\text{inside}} &= \frac{-2.0\mu\text{C}}{4\pi(0.040\text{m})^2} = -1.0\times10^{-4}\frac{\text{C}}{\text{m}^2} \\ \sigma_{\text{outside}} &= \frac{12.0\mu\text{C}}{4\pi(0.050\text{m})^2} = 3.8\times10^{-4}\frac{\text{C}}{\text{m}^2} \end{aligned}$$

2.8 $C = \frac{Q}{\Delta V} \ \longrightarrow\ Q = C\ \Delta V = (6.0\mu\text{F})(1.5\text{V}) = 9.0\mu\text{C}$.

2.9 From Gauss's law the electric field near a charged plate is $\sigma/2\epsilon_0$. Since there are two plates, the field between the plates is $E = \sigma/\epsilon_0$. Also the electric field is related to the electric potential difference between the plates, $E = \Delta V/d$. Thus we find that

$$\frac{\sigma}{\epsilon_0} = \frac{\Delta V}{d} \ \longrightarrow\ \frac{\sigma}{\Delta V} = \frac{\epsilon_0}{d}$$

But the charge on the plates is $Q = \sigma A$ so that

$$C = \frac{Q}{\Delta V} = \frac{\sigma A}{\Delta V} = \frac{\sigma}{\Delta V}A = \frac{\epsilon_0}{d}A = \frac{\epsilon_0 A}{d}$$

2.10 Assume that the capacitor is charged so that the inside sphere has a charge $-Q$ and the outside sphere has a charge $+Q$. By Gauss's law the field between the shells is $E4\pi r^2 = -Q/\epsilon_0 \ \longrightarrow\ -Q/4\pi\epsilon_0 r^2$ Thus we can find the electric potential difference

$$\Delta V = -\int_a^b E\ dr = -\int_a^b \frac{-Q}{4\pi\epsilon_0 r^2}dr = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Thus

$$C = \frac{Q}{\Delta V} = \frac{4\pi\epsilon_0}{1/a - 1/b}$$

2.11 First suppose that there is a charge Q on the length L of the central wire, and a charge $-Q$ on the outer shield. Now imagine a gaussian surface around the central wire that is between the outer shield and the wire, and a distance r from the wire. There is no flux through the ends of this surface and the field is normal to the cylindrical surface. Thus the net flux is $\phi = EA$ where A is the area of the surface $2\pi rL$. Thus $\phi = \frac{Q_{\text{in}}}{\epsilon_0} \longrightarrow E = \frac{Q}{2\pi L\epsilon_0}\frac{1}{r}$ We want to find the potential difference between the wire and shield.

$$\begin{aligned}\Delta V &= V_a - V_b \\ &= -\int_b^a \vec{E}\cdot d\vec{r} \\ &= \int_a^b \vec{E}\cdot d\vec{r} \\ &= \int_a^b E\ dr \\ &= \int_a^b \frac{Q}{2\pi L\epsilon_0}\frac{1}{r}\ dr \\ &= \frac{Q}{2\pi L\epsilon_0}\left[\ln(r)\right]_a^b \\ &= \frac{Q}{2\pi L\epsilon_0}\left[\ln(b) - \ln(a)\right] \\ &= \frac{Q}{2\pi L\epsilon_0}\ln(b/a)\end{aligned}$$

Now we can us this to find that

$$C = \frac{Q}{\Delta V} = \frac{2\pi\epsilon_0 L}{\ln(b/a)}$$

2.12 Move from the negatively charged wire to the positively charged wire. Find the change in potential due to each wire and then add. From Gauss's law we know that the electric field due to the positive wire is given by $E = \frac{\lambda}{2\pi\epsilon_o r}$. Thus the change in potential due to the positive wire is

$$\Delta V_+ = -\int_{b-a}^{a} E_+ dr = -\int_{b-a}^{a}\frac{\lambda dr}{2\pi\epsilon_o r} = \frac{\lambda}{2\pi\epsilon_o}\ln\left(\tfrac{b-a}{a}\right)$$

Similarly the change in potential due to the negative wire is

$$\Delta V_+ = -\int_a^{b-a} E_- dr = -\int_a^{b-a}\frac{-\lambda dr}{2\pi\epsilon_o r} = \frac{\lambda}{2\pi\epsilon_o}\ln\left(\tfrac{b-a}{a}\right)$$

Thus the total change in electric potential in going from the negative to the positive wire is

$$V = \Delta V_+ + \Delta V_- = \frac{\lambda}{\pi\epsilon_o}\ln\left(\tfrac{b-a}{a}\right)$$

Putting $\lambda = Q/\ell$ in the above equation and solving for Q we find

$$Q = \ell\frac{\pi\epsilon_o}{\ln\left(\frac{b-a}{a}\right)}V \longrightarrow C = \ell\frac{\pi\epsilon_o}{\ln\left(\frac{b-a}{a}\right)} \longrightarrow \frac{C}{\ell} = \frac{\pi\epsilon_o}{\ln\left(\frac{b-a}{a}\right)}$$

2.13
(a) $U = \frac{1}{2}CV_C^2 = \frac{1}{2}(120\mu\text{F})(100\text{V})^2 = 0.6\text{J}$.
(b) Since there is a maximum field strength there is also a maximum energy density $u_{\max} = \frac{1}{2}\epsilon_0 E_{\max}^2 = \frac{1}{2}\epsilon_0(3\times10^6\,\frac{\text{V}}{\text{m}})^2 = 39.8\text{J/m}^3$. So $\frac{U}{\mathcal{V}} < u_{\max} \longrightarrow \mathcal{V} > \frac{U}{u_{\max}} = \frac{0.6\text{J}}{39.8\text{J/m}^3} = 0.015\text{m}^3$.

2.14 With $q < 0$ the electric field is pointed toward the center and $q = -|q|$ so that

$$\vec{E} = \frac{|q|}{4\pi\epsilon_0 r^2}(-\hat{r}) = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}$$

But this is the same as form as for the positive particle so the integral will also be of the same form.

2.15

$$\begin{aligned}&\Delta K + \Delta U = 0 \\ \longrightarrow\ & K_f - 0 = -\Delta U = -q\Delta V = e\Delta V \\ & \tfrac{1}{2}mv_f^2 = e\frac{e}{4\pi\epsilon_0}\left[\frac{1}{r_f} - \frac{1}{r_i}\right] \\ \longrightarrow\ & v_f = \sqrt{\frac{2}{m}\frac{e^2}{4\pi\epsilon_0}\left[\frac{1}{0.005\text{m}} - \frac{1}{0.02\text{m}}\right]} = 275\,\frac{\text{m}}{\text{s}}\end{aligned}$$

2.16 $\Delta K + \Delta U = W_{\text{nc}} \longrightarrow (\frac{1}{2}mv_f^2 - 0) + q\Delta V = 0$. Solving for the potential difference we find

$$\Delta V = -mv_f^2/2q = m(0.4c)^2/2e = 41\text{kV}.$$

2.17 $\Delta K + \Delta U = W_{\text{nc}} \longrightarrow 0 + q\Delta V = W_{\text{nc}}$. But $q = N_A(-e)$ and $\Delta V = V_f - V_i = -14\text{V}$ so $W_{\text{nc}} = 1.35\times10^6\text{J}$.

2.18 $\Delta K + \Delta U = W_{\text{nc}} \longrightarrow (\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2) + q\Delta V = 0$ Thus $\Delta V = -(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2)/q = -38.9\text{V}$. We see that the final point is at a lower electric potential. Note also that this depends in no way on the distance 2.0 cm.

2.19 For the alpha particle $q = 2e$ and

$$\Delta K + \Delta U = W_{\text{nc}} \longrightarrow (0 - \tfrac{1}{2}mv_i^2) + q\Delta V = 0.$$

Now consider the electric potential difference ΔV. This is due to the change in position relative to the nuclear charge $Q = 79e$. Thus

$$\Delta V = V_f - V_i = k\frac{Q}{r_f} - k\frac{Q}{r_i} = k\frac{Q}{r_f} - k\frac{Q}{\infty} = k\frac{Q}{r_f}$$

Putting this expression of the electric potential difference into the previous equation and solving for r_f we find

$$r_f = \frac{2kqQ}{mv_i^2} = 2.8 \times 10^{-14}\text{m}$$

Note: The alpha particles were not really fired at a gold nucleus but rather at a thin sheet of gold. Thus since the gold atoms are neutral there is no electrical force until the alpha particle passes inside the atom. It would thus have been a better approximation to take to potential difference between the atomic radius and the stopping radius. This would be equivalent to presuming that the electrons all reside on the surface of the atom. The cruder approximation that we actually did gives a good result anyway since the atomic size it about four orders of magnitude larger then the stopping radius, and thus is effectively at infinity anyway. It is strange to think that an atomic radius could be effectively infinity. This gives us an idea of how small the nucleus is compared with the atom.

2.20 Let us bring the charges in one at a time and find the work done to bring each one. The work to bring the first is zero since there is no repulsive force to overcome, $W_1 = 0$. The work to bring in the second is the change in potential energy of the second charge in the field of the first as we bring the second charge in from far away to a distance s from the first. Thus

$$W_2 = Q\Delta V_2 = Q\left(\frac{kQ}{r_f} - \frac{kQ}{r_i}\right) = \frac{kQ^2}{s}.$$

Now we have two charges so the third charge will have a change in potential due to both of the first two charges.

So $$W_3 = Q\Delta V_3 = \tfrac{kQ^2}{s} + \tfrac{kQ^2}{\sqrt{2}s}.$$

Similarly $$W_4 = Q\Delta V_4 = \tfrac{kQ^2}{s} + \tfrac{kQ^2}{s} + \tfrac{kQ^2}{\sqrt{2}s}.$$

So the total work is $$W = \left(4 + \tfrac{2}{\sqrt{2}}\right)\tfrac{kQ^2}{s} = \left(4 + \sqrt{2}\right)\tfrac{kQ^2}{s}\ .$$

2.21 We know that $\vec{E} = -\vec{\nabla}V$ so

$$\begin{aligned}\vec{E} &= -\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} - \frac{\partial V}{\partial z}\hat{k} \\ &= (-5 + 6xy)\hat{\imath} + (3x^2 - 2z^2)\hat{\jmath} + (-4yz)\hat{k} = -5\hat{\imath} - 5\hat{\jmath} + 0\hat{k}\end{aligned}$$

2.22 The constant α must have the units of charge per area.

$$\begin{aligned}V = \int k\frac{dq}{r} &= \int_0^L k\frac{\lambda dx}{r} = \int_0^L k\frac{\alpha x dx}{x+d} \\ &= \alpha k[x + d - d\ln(x+d)]_0^L \\ &= \alpha k[L - d\ln(1 + L/d)]\end{aligned}$$

2.23 The electric potential due to the curved section is

$$V_c = \int k\frac{dq}{r} = \int_0^{\pi R} k\frac{\lambda ds}{R} = k\frac{\lambda\pi R}{R} = k\lambda\pi.$$

The electric potential due to one of the straight sections is

$$V_s = \int k\frac{dq}{r} = \int_R^{3R} k\frac{\lambda dx}{x} = k\lambda\ln\frac{3R}{R} = k\lambda\ln 3.$$

The total electric potential is

$$V = 2V_s + V_c = k\lambda(2\ln 3 + \pi).$$

2.24 We know that for a point charge $E = kq/r^2$ and $V = kq/r$. Thus $V/E = r$ and we can find that $r = 6.0$m. We can then solve $V = kq/r$ for q and find $q = Vr/k = 2.0\mu$C.

2.25

(a)

$$V = \frac{kQ}{x+d} + \frac{kQ}{x-d} + \frac{k(-2Q)}{x} = \frac{kQ2x}{x^2-d^2} + \frac{k(-2Q)}{x} = \frac{2kQd^2}{x(x^2-d^2)}$$

(b) By the placement of the charges we know that on the x-axis the field is parallel to the x-axis:

$$\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k} = E_x\hat{\imath} + 0\hat{\jmath} + 0\hat{k} = E_x\hat{\imath}.$$

But also we know that

$$E_x = -\frac{\partial V}{\partial x} = \frac{2kQd^2(3x^2-d^2)}{x^2(x^2-d^2)^2}.$$

(c) Since $x \gg d$ we can ignore any d^n that is summed with an x^n.

$$V \approx \frac{2kQd^2}{x(x^2-0)} = \frac{2kQd^2}{x^3}$$

$$E_x \approx \frac{2kQd^2(3x^2-0)}{x^2(x^2-0)^2} = \frac{6kQd^2}{x^4}$$

2.26 First let us find two other relationships; charge and radius. The charge (Q) on the large drop is twice the charge (Q_o) on the smaller drops:

$$Q = 2Q_o.$$

Also the volume of the large drop is twice the volume of the smaller drops so $\frac{4}{3}\pi r^3 = 2\frac{4}{3}\pi r_o^3$ so

$$r = 2^{1/3}r_o.$$

Now we can find the surface density

$$\sigma = \frac{Q}{A} = \frac{Q}{4\pi r^2} = \frac{2Q_o}{4\pi(2^{1/3}r_o)^2} = 2^{1/3}\frac{Q_o}{4\pi r_o^2} = 2^{1/3}\sigma_o\ ,$$

and the field strength

$$E = k\frac{Q}{r^2} = k\frac{2Q_o}{(2^{1/3}r_o)^2} = 2^{1/3}k\frac{Q_o}{r_o^2} = 2^{1/3}E_o\ ,$$

and the potential

$$V = k\frac{Q}{r} = k\frac{2Q_o}{2^{1/3}r_o} = 2^{2/3}k\frac{Q_o}{r_o} = 2^{2/3}V_o\ .$$

2.27 Suppose that we are charging up the sphere slowly and that right now the sphere has built up a charge q. Now consider how much work would be required to bring an additional charge dq from infinitely far away to the surface of the sphere. $dW = dU = dq\ \Delta V = dq(V_R - V_\infty) = dq(k\frac{q}{R} - 0) = k\frac{q\ dq}{R}$. Now we can add up the work done to bring each bit of charge as the sphere was charged up from zero to Q.

$$W = \int dW = \int_0^Q k\frac{q\ dq}{R} = \tfrac{1}{2}k\frac{Q^2}{R}$$

2.28 By the symmetry of the charge distribution we know that the field is radial at all places so we need only find the strength. By Gauss's law we know that the field is $E = k\frac{q_{\text{in}}}{r^2}$. Thus with $q_1 = 10\text{nC}$ and $q_2 = -15\text{nC}$

$$E = \begin{cases} 0 & \text{for} \quad r < a \\ k\frac{q_1}{r^2} & \text{for } a < r < b \\ k\frac{q_1+q_2}{r^2} & \text{for } b < r \end{cases}$$

With the E-field we can now find the electric potential via

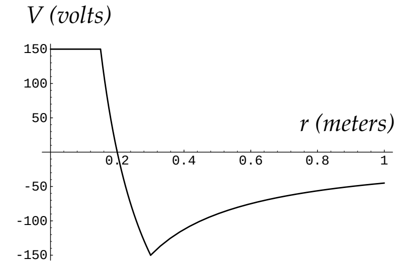
$$V = V - V_\infty = -\int_\infty^r E dr$$

If $r > b$ then $E = k\frac{q_1+q_2}{r^2}$ and the integral gives $V = k\frac{q_1+q_2}{r}$. If $a < r < b$ then we must split the integral into two parts

$$\begin{aligned} V &= -\int_\infty^r E dr = -\left(\int_\infty^b E dr + \int_b^r E dr\right) \\ &= -\int_\infty^b k\frac{q_1+q_2}{r^2}dr - \int_b^r k\frac{q_1}{r^2}dr \\ &= \left[k\frac{q_1+q_2}{b} - 0\right] + \left[k\frac{q_1}{r} - k\frac{q_1}{b}\right] = k\frac{q_2}{b} + k\frac{q_1}{r} \end{aligned}$$

If $r < a$ then we must split the integral into three parts

$$\begin{aligned} V &= -\int_\infty^b E dr - \int_b^a E dr - \int_a^r E dr \\ &= -\int_\infty^b E dr - \int_b^a E dr - \int_a^r 0 dr = k\frac{q_2}{b} + k\frac{q_1}{a} \end{aligned}$$



2.29 $Q = CV \longrightarrow C = Q/C = 1.0\mu\text{F}$. The charge and voltage are proportional so since the charge has increased ten times the voltage must increase ten time also so $V = 100\text{V}$.

2.30 $Q = CV \longrightarrow C = Q/C = Ne/C = 18.0\text{nF}$

2.31 The energy density of the field is $\frac{1}{2}\epsilon_o E^2$ so the total energy is

$$U = \int \tfrac{1}{2}\epsilon_o E^2 d\mathcal{V}.$$

But we know that $E = kQ/r^2$ for $r > R$ and zero for $r < R$ and also we know that the volume of a spherical shell of thickness dr is $d\mathcal{V} = 4\pi r^2 dr$ so

$$U = \int_R^\infty \tfrac{1}{2}\epsilon_o\left(\frac{kQ}{r^2}\right)^2 4\pi r^2 dr = \tfrac{1}{2}4\pi\epsilon_o k^2 Q^2 \int_R^\infty \frac{dr}{r^2} = \frac{kQ^2}{2R}$$

Equating this to mc^2, solving for R and setting $Q = e$ we find

$$R = \frac{ke^2}{mc^2} = 2.8 \times 10^{-15}\text{m}.$$

3.1
(a) The amount of charge that passes through can be found from the definition of current.

$$dq = I dt$$

This amount of charge is also equal to the number of electrons that pass through dN times the charge of an electron e.

$$dq = dN\ e$$

Combining these to eliminate dq we find

$$dN = \frac{Idt}{e} = \frac{(1.0 \times 10^{-3}\text{C/s})(1.6 \times 10^{2}\text{s})}{1.6 \times 10^{-19}\text{C}} = 10^{18}$$

(b)

$$J = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{(1.0 \times 10^{-3}\text{C/s})}{\pi(2.0 \times 10^{-4}\text{m})^2} = 8.0 \times 10^{3}\text{A/m}^2$$

3.2 Suppose that the electric potential difference between the ends of the wire is ΔV, and the current through the wire is I. We can relate the electric potential difference to the electric field strength.

$$\Delta V = E\ dr = EL$$

We can use Ohm's law to relate the electric field to the current density.

$$J = \frac{1}{\rho}E$$

Combining these we find

$$\Delta V = \rho JL$$

But the current density is $J = I/A$ so that we can write

$$\Delta V = \frac{\rho IL}{A}$$

From this we can compute the resistance.

$$R = \frac{\Delta V}{I} = \rho\frac{L}{A}$$

3.3 The greatest resistance will be found by picking the two sides that are furthest apart ($3a$). For these the cross sectional area is $2a \times a = 2a^2$ and so

$$R = \rho\frac{L}{A} = \rho\frac{3a}{2a^2} = \rho\frac{3}{2a}$$

The least resistance will be found by picking the two sides that are closest together (a). For these the cross sectional area is $2a \times 3a = 6a^2$ and so

$$R = \rho\frac{L}{A} = \rho\frac{a}{6a^2} = \rho\frac{1}{6a}$$

3.4 $P = I\Delta V \longrightarrow I = P/\Delta V = 0.5\text{A}$.

3.5
(a) $P = I\ \Delta V = I(IR) = I^2R$.
(b) $P = I\ \Delta V = (\Delta V/R)\Delta V = (\Delta V)^2/R$.

3.6

$$V_C = V_S(1 - e^{-t/RC}) \longrightarrow e^{-t/RC} = 1 - V_C/V_S = 1 - 5/10 = 1/2$$
$$\longrightarrow -t/RC = \ln(1/2) \longrightarrow t = -RC\ln(1/2) = 10.39\text{ms}$$

3.7 Going around counter clockwise, Kirchhoff's loop rule gives us

$$V_C + V_R = 0 \longrightarrow V_R = -V_C$$

But since $Q = CV_C$ we know that $I = dQ/dt = CdV_C/dt$ and

$$V_R = IR = RC\frac{dV_C}{dt}$$

Putting this into $V_R = -V_C$ we find that

$$RC\frac{dV_C}{dt} = -V_C \longrightarrow \frac{dV_C}{dt} = -\frac{1}{RC}V_C$$

Now we can check our proposed solution $V_C = V_S e^{-t/RC}$ to see if it satisfies this differential equation.

$$V_C = V_S e^{-t/RC} \longrightarrow \frac{dV_C}{dt} = V_S e^{-t/RC}\left(-\frac{1}{RC}\right) = -\frac{1}{RC}V_C$$

So we see that this does satisfy the differential equation.

3.8 $\Sigma F = ma \rightarrow k\frac{e^2}{r^2} = m\frac{v^2}{r} \rightarrow v = \sqrt{\frac{k}{mr}}e = 2.19 \times 10^6\,\frac{\text{m}}{\text{s}}$. The time for one orbit is $\Delta t = 2\pi r/v = 1.52 \times 10^{-16}$s. So $I = \Delta Q/\Delta t = e/\Delta t = 1.05$mA.

3.9 $I = \Delta Q/\Delta t = Ne/\Delta t \longrightarrow N = I\Delta t/e = 7.5 \times 10^{15}$.

3.10 $I = dq/dt = 12t^2 + 5 = 17\text{A}$and $J = I/A = 8.5\text{A/cm}^2$.

3.11 Let $I_o = 100.0$A and $\omega = 120\pi\text{s}^{-1}$. Then $I = I_o \sin\omega t$ and

$$\Delta q = \int_0^{\Delta t}\frac{dq}{dt}dt = \int_0^{\Delta t} I_o\sin\omega t dt = \left[-\frac{I_o}{\omega}\cos\omega t\right]_0^{\Delta t} = \frac{I_o}{\omega}$$

So $\Delta q = 0.265$C.

3.12 If M is the mass of the wire and $m = \frac{63.54\text{g}}{N_A}$ is the mass of one atom then the charge in a length ℓ of wire is

$$Q = Ne = \frac{M}{m}e = \frac{\rho\ell A}{m}e.$$

The amount of charge that passes a point in the wire in a time Δt is $Q = I\Delta t$. So we can say that a length ℓ of the charge passes in a time Δt if

$$\frac{\rho\ell A}{m}e = Q = I\Delta t$$

But this would imply that the charges are moving with a speed of $v = \ell/\Delta t$. So

$$v = \frac{\ell}{\Delta t} = \frac{mI}{\rho A e} = 7.4 \times 10^{-5}\,\frac{\text{m}}{\text{s}}$$

3.13 $R = \rho\frac{\ell}{A} = \rho\frac{\ell}{\pi r^2} = 0.310\Omega$.

3.14 $P = IV = V^2/R$ so $\frac{P}{P_o} = \frac{V^2/R}{V_o^2/R} = 1.36 \longrightarrow 36\%$.

3.15 Since the mass is the same the volume must be the same so

$$V_A = V_B \longrightarrow L_A A_A = L_B A_B \longrightarrow \frac{A_B}{A_A} = \frac{L_A}{L_B} = 2$$

Also, the resistance of a wire is proportional to the ratio of the length and cross-sectional area. So that

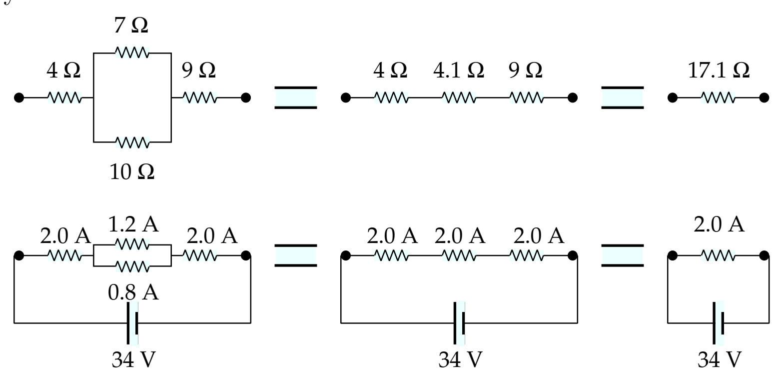
$$\frac{R_A}{R_B} = \frac{L_A/A_A}{L_B/A_B} = \frac{L_A}{L_B}\frac{A_B}{A_A} = 2 \cdot 2 = 4$$

3.16 $1/A \cdot 1\text{hr} = 3600\text{C}$ so $55\text{A} \cdot \text{hr} = 198,000\text{C}$. Thus the potential energy is $U = QV = (198,000\text{C})(12V) = 2376\text{kJ}$. One kilowatt hour is really a unit of energy 3600kJ. So if 3600kJ costs 12 cents 2376kJ will cost about 8 cents. Not much!

3.17 $cm\Delta T = \text{heat} = P\Delta t = \frac{V^2}{R}\Delta t \rightarrow R = \frac{V^2\Delta t}{cm\Delta T} = 29\Omega$.

3.18 The voltage on the terminals of the battery is $V = \mathcal{E} - Ir$ but this is also the voltage on the resistor $V = IR$ so that $IR = \mathcal{E} - Ir$.
(a) From this we can find that $R = \mathcal{E}/I - r = 7.7\Omega$.
(b) The power lost in the internal resistor is $P = IV = I^2 r = 1.7\text{W}$.

3.19 First find the equivalent resistance and then connect it to the 34 V power supply and find the currents.



(a) The 7Ω and 10Ω resistors are in parallel so they can be combined into a 4.1Ω resistor. This is in series with the other two resistors so that the effective resistance of the system is $R_{\text{eff}} = 4\Omega + 4.1\Omega + 9\Omega = 17.1\Omega$.

(b) The current through the system will be $(34\text{V})/(17.1\Omega) = 2.0\text{A}$. This will also be the current through the 4Ω and 9Ω resistors. The other two resistors share the current, $I = I_1 + I_2$. Since these two resistors are in parallel we know that the voltage on them is the same so that $I_1 R_1 = I_2 R_2$. Combining these two equations we find

$$I_1 = \frac{R_2}{R_1 + R_2} I.$$

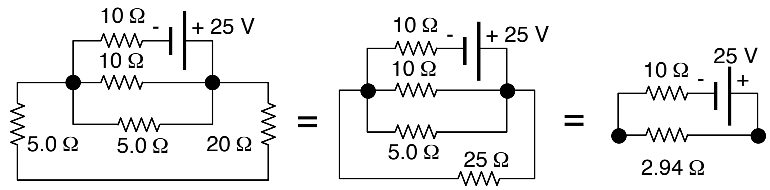
This result will work any time two resistors are in parallel. In this particular case

$$I_{7\Omega} = \frac{10}{7 + 10}(2.0\text{A}) = 1.2\text{A}$$

So

$$I_{10\Omega} = I - I_{7\Omega} = 0.8\text{A}$$

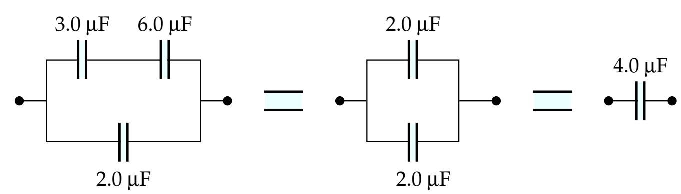
3.20 First lump the resistors and between points a and b



Now we see that the voltage between a and b is $V_{ab} = \frac{2.94\Omega}{10\Omega + 2.94\Omega} 25\text{V} = 5.68\text{V}$. But the 20Ω and the 5Ω are in series across this 5.68 V so the current through these resistors is $I = 5.68\text{V}/(20\Omega + 5\Omega) = 0.227\text{A}$.

3.21 Adding the two that are in series we find that

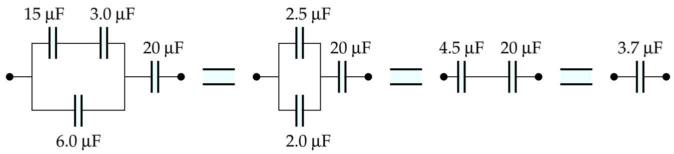
$$\frac{1}{C_a} = \frac{1}{3.0\mu\text{F}} + \frac{1}{6.0\mu\text{F}} \longrightarrow C_a = 2.0\mu\text{F}.$$



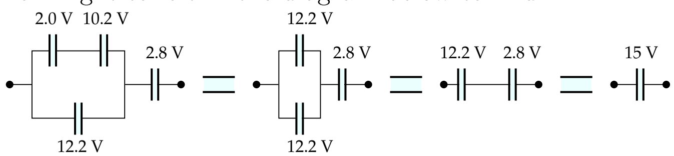
This is in parallel with the real 2.0 μF capacitor so the net capacitance is

$$C_{\text{eff}} = 2.0\mu\text{F} + 2.0\mu\text{F} = 4.0\mu\text{F}.$$

3.22 As the last problem we find the effective capacitance by the divide and conquer method.



Now to figure the voltages we first consider the general problem of two capacitors in series with a voltage V across the pair. We know that since they are in series they must have the same charge so $Q_1 = Q_2 \longrightarrow C_1V_1 = C_2V_2$ but also we know that the total voltage is the sum of the individual voltages so that $V = V_1 + V_2$ Combining these two equations in order to eliminate V_2 and then solving for V_1 we find $V_1 = \frac{V}{1+C_1/C_2}$, and similarly $V_2 = \frac{V}{1+C_2/C_1}$. With this result and the observation that capacitors in parallel have the same voltage we can work from right to left in the diagram below to find.



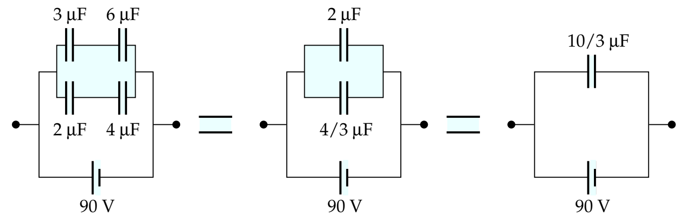
3.23 In parallel the capacitance is $C_p = C_1 + C_2 = 4.00\mu$F. In series the capacitance is given by $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$ and also equal to one quarter the individual capacitances of one of the capacitors $C_s = \frac{1}{4}C_1$. Combining these last two equations we find

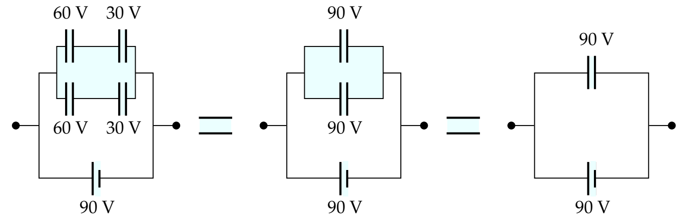
$$\frac{4}{C_1} = \frac{1}{C_1} + \frac{1}{C_2} \longrightarrow C_1 = 3C_2$$

Putting this result into the first equation we find

$$C_p = 3C_2 + C_2 = 4.00\mu\text{F} \longrightarrow C_2 = 1.00\mu\text{F and } C_1 = 3.00\mu\text{F}$$

3.24 Divide and conquer again!



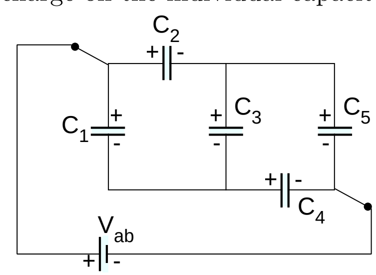


With the voltages we can easily find the charges using $Q = CV$

$$Q_2 = C_2V_2 = 2\mu\text{F} \cdot 60\text{V} = 120\mu\text{C}$$
$$Q_3 = C_3V_3 = 3\mu\text{F} \cdot 60\text{V} = 180\mu\text{C}$$
$$Q_4 = C_4V_4 = 4\mu\text{F} \cdot 30\text{V} = 120\mu\text{C}$$
$$Q_6 = C_6V_6 = 6\mu\text{F} \cdot 30\text{V} = 180\mu\text{C}$$

$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = 13.5$mJ

3.25 Let us first add a power supply to charge up the capacitors and find the voltages and thus the charge on the individual capacitors.



Notice that the charge for C_4 comes from C_1 and C_3, so that

$$Q_4 = Q_1 + Q_3 \longrightarrow C_4V_4 = C_1V_1 + C_3V_3$$

which with these particular capacitors becomes

$$V_4 = \frac{C_1}{C_4}V_1 + \frac{C_3}{C_4}V_3 \longrightarrow V_4 = V_1 + 4V_3.$$

Similarly

$$Q_2 = Q_3 + Q_5 \longrightarrow V_2 = 2V_3 + V_5$$

We can find three other equations with Kirchhoff's loop rule

$$\begin{aligned} V_1 - V_2 - V_3 &= 0 \\ V_3 - V_5 + V_4 &= 0 \\ V_{ab} - V_1 - V_4 &= 0 \end{aligned} \quad \text{plus the two above} \quad \begin{aligned} V_4 &= V_1 + 4V_3 \\ V_2 &= 2V_3 + V_5 \end{aligned}$$

Using the second equation to eliminate V_5 we find

$$\begin{aligned} V_1 - V_2 - V_3 &= 0 & V_4 &= V_1 + 4V_3 \\ V_{ab} - V_1 - V_4 &= 0 & V_2 &= 2V_3 + (V_3 + V_4) = 3V_3 + V_4 \end{aligned}$$

Using the second to eliminate V_4 we find

$$V_1 - V_2 - V_3 = 0 \quad \begin{aligned} V_{ab} &= 2V_1 + 4V_3 \\ V_2 &= 3V_3 + (V_{ab} - V_1) \end{aligned}$$

Using the first to eliminate V_3 we find

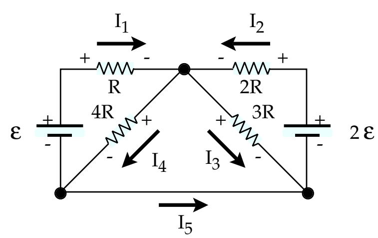
$$\begin{aligned} V_{ab} &= 6V_1 - 4V_2 \\ 4V_2 &= 2V_1 + V_{ab} \end{aligned}$$

These two equations give us $V_1 = V_2 = \frac{1}{2}V_{ab}$. Now notice that the total charge supplied by the battery is is

$$Q = Q_1 + Q_2 = C_1V_1 + C_2V_2 = \tfrac{1}{2}(C_1 + C_2)V_{ab}$$

Thus we find that the capacitance between points a and b is $C = \frac{1}{2}(C_1 + C_2) = 3.0\mu\text{F}$.

3.26 First let us label the currents and show the direction of potential difference on the resistors. Now we can write out the two junction rules and the three loop rules.



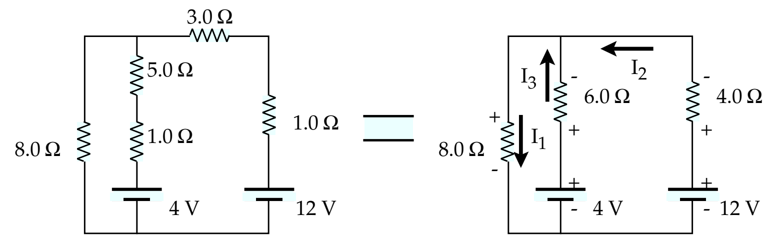
$$\begin{aligned} I_4 &= I_1 + I_5 \\ I_3 + I_5 &= I_2 \\ \mathcal{E} - I_1R - I_44R &= 0 \\ 2\mathcal{E} - I_22R - I_33R &= 0 \\ I_44R - I_33R &= 0 \end{aligned} \quad \underset{I_o = \frac{\mathcal{E}}{R}}{\longrightarrow} \quad \begin{aligned} I_4 &= I_1 + I_5 \\ I_3 + I_5 &= I_2 \\ I_o - I_1 - I_44 &= 0 \\ 2I_o - I_22 - I_33 &= 0 \\ I_44 - I_33 &= 0 \end{aligned}$$

Using the first two we can eliminate I_2 and I_4 we get

$$\begin{aligned} I_o - 5I_1 - 4I_5 &= 0 \\ 2I_o - 5I_3 - 2I_5 &= 0 \\ 4I_1 + 4I_5 - 3I_3 &= 0 \end{aligned} \longrightarrow \begin{aligned} I_o - 5I_1 - 4I_5 &= 0 \\ 3I_o - 10I_1 - 13I_5 &= 0 \end{aligned} \longrightarrow I_5 = \frac{I_o}{5}$$

The last of the resulting three equations was used to eliminate I_3. After which the two were combine to eliminate I_1. Thus we find $I_5 = 50\text{mA}$. Since this is positive we know that the direction of the arrow is the direction of the current.

3.27 First let us combine resistors and label the currents.



The junction and two current equations are

$$\begin{aligned} I_1 &= I_2 + I_3 & & & I_1 &= I_2 + I_3 \\ I_18\Omega + I_36\Omega - 4\text{V} &= 0 & &\longrightarrow & I_14 + I_33 &= 2\text{A} \\ I_18\Omega + I_24\Omega - 12\text{V} &= 0 & & & I_12 + I_2 &= 3\text{A} \end{aligned}$$

Now use the first to eliminate I_1 and then eliminate I_2 to find

$$\begin{aligned} 4I_2 + 7I_3 &= 2\text{A} \\ 3I_2 + 2I_3 &= 3\text{A} \end{aligned} \longrightarrow 13I_3 = -6\text{A} \longrightarrow \begin{aligned} I_1 &= \tfrac{11}{13}\text{A} \\ I_2 &= \tfrac{17}{13}\text{A} \\ I_3 &= -\tfrac{6}{13}\text{A} \end{aligned}$$

3.28 Assume the resistance of the two light bulbs is constant. Then for the same potential difference, the 25W light bulb has less current than the 100W light bulb since $P = IV$. Since $R = V/I$, $R_{25} > R_{100}$. The intensity of a light bulb is proportional to the power being dissipated, and this is equal to I^2R. So, put the light bulbs in series; the current though each will be the same (different potential differences). If the current is the same, then the bulb with the greatest R will have the greatest I^2R. Since R_{25} is larger, the 25W bulb is brightest.

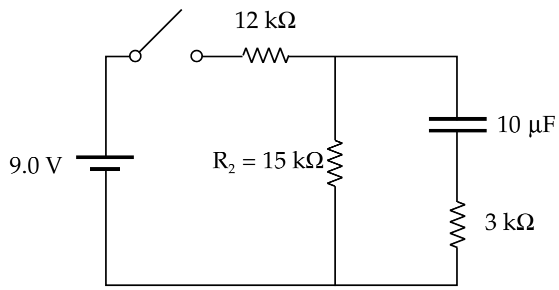
3.29 $P = IV \longrightarrow I = P/V$ so $I_{1500} = 12.5\text{A}$ and $I_{1000} = 8.33\text{A}$ and $I_{750} = 6.25\text{A}$. Thus the total current drawn is 27.08 A, and more then the circuit can handle.

3.30 In a previous problem we found the resistance of 15m of 12 gauge copper wire to be 0.310Ω. Thus 16 feet will have a resistance of $R = 0.1\Omega$, and the power lost in the wire will be $P_{1.0\text{A}} = I^2R = 0.1\text{W}$ and $P_{10\text{A}} = I^2R = 10\text{W}$.

3.31 Let us write the power in terms of the characteristics of the wire $P = I^2R = I^2\rho\ell/A \longrightarrow P/\ell = I^2\rho/A$ Thus if the two wires are to have the same power per length at maximum current we relate the maximum currents by

$$I_{Al}^2\frac{\rho_{Al}}{A} = I_{Cu}^2\frac{\rho_{Cu}}{A} \longrightarrow I_{Al} = I_{Cu}\sqrt{\frac{\rho_{Cu}}{\rho_{Al}}} = 15.5\text{A}$$

3.32 Once steady state is reached there is no charge flowing to the capacitor and thus no current in the 3 kΩ resistor. With no current through that branch we know that the current in the 12 and 15 kΩ resistors must be the same and equal to $9.0\text{V}/(12\text{k}\Omega + 15\text{k}\Omega) = \frac{1}{3}\text{mA}$



Since there is no current running through the 3kΩ resistor the voltage on the capacitor will be the same as the voltage (5V) on the 15kΩ resistor. Thus the initial charge on the capacitor is $q_o = C(5\text{V}) = 50\mu\text{C}$

When the switch is opened the 12kΩ resistor and the battery are effectively disconnected from the circuit. Thus we end up with an effective resistance of $R = 15\text{k}\Omega + 3.0\text{k}\Omega$ in a loop with a capacitor with a charge of $q_o = C(5\text{V}) = 50\mu\text{C}$. The charge on the capacitor will drop exponentially as $q = q_oe^{-t/RC}$. This will cause a current

$$I = -dq/dt = (q_o/RC)e^{-t/RC} = I_oe^{-t/RC}$$

with $I_o = 278\mu\text{A}$ and $RC = 0.18\text{s}$

If $I = I_o/5$ then $e^{-t/RC} = 1/5 \longrightarrow t = RC\ln 5 = 0.29\text{s}$.

4.1
(a) The vector $\vec{r} = R\cos\theta\hat{\imath} + R\sin\theta\hat{\jmath}$ points to the wire at the location θ. Thus if we move a small angle $d\theta$ the position vector moves a small amount

$$\vec{dr} = \frac{d\vec{r}}{d\theta}d\theta = (-R\sin\theta\hat{\imath} + R\cos\theta\hat{\jmath})d\theta = (-\sin\theta\hat{\imath} + \cos\theta\hat{\jmath})Rd\theta$$

But the change in the position is the vector $d\ell$ that we are looking for.
(b)

$$\vec{d\ell} \times \vec{B} = (-\sin\theta\hat{\imath} + \cos\theta\hat{\jmath})Rd\theta \times B\hat{\jmath} = -\sin\theta B\ Rd\theta\hat{k}$$

So

$$\vec{F} = \int I\ \vec{d\ell} \times \vec{B} = -IBR\hat{k}\int_0^\pi \sin\theta d\theta = -2IBR\hat{k}$$

(c) $\vec{\Delta\ell} = -2R\hat{\imath}$ So $\vec{F} = I\vec{\Delta\ell} \times \vec{B} = I(-2R\hat{\imath}) \times B\hat{\jmath} = -2IBR\hat{k}$.

4.2 The force on the two side pieces is zero because the current is parallel to the field. Using the RHR we see that the force on the upper section is out of the paper, so that the torque is to the right. The magnitude of the force on the upper wire is $I\ \Delta\ell\ B = IwB$. This force is at a distance $h/2$ from the axis of rotation so it creates a torque $\vec{\tau}_{\text{upper}} = \vec{r} \times \vec{F} = \frac{h}{2}IwB\hat{\imath}$. The force on the lower section is also IwB but is into the paper, which also gives a torque to the right. The the net torque is the sum of these two torques $\tau = \tau_{\text{upper}} + \tau_{\text{lower}} = \frac{h}{2}IwB + \frac{h}{2}IwB = IhwB$. Note that hw is the area of the loop so that the torque is IAB, the product of the current, area and field strength. This ends up being true regardless of the shape of the loop, when the field is parallel to the plane of the loop.

4.3 $qvB = m\frac{v^2}{r} \longrightarrow m = qB\frac{r}{v} = 9.19 \times 10^{-31}\text{kg}$.

4.4
(a) A,B, and C.
(b) C or D.
(c) C or D.
(d) A or F.

4.5 It will take a time equal to the circumference divided by the velocity to complete one revolution.

$$T = \frac{2\pi r}{v} = 2\pi\frac{r}{v}$$

But

$$F = ma \longrightarrow qvB = m\frac{v^2}{r} \longrightarrow \frac{r}{v} = \frac{m}{qB}$$

so that

$$T = 2\pi\frac{m}{qB}$$

We see then that the time it takes to make one revolution depends only on the mass, and charge of the particle and the magnetic field strength. It does not depend on the velocity of the particle.

4.6 First let us compute the Lorentz force.

$$\begin{aligned}\vec{F} &= q(\vec{E} + \vec{v} \times \vec{B})\\ &= q(E\hat{\jmath} + v\hat{\imath} \times B\hat{k})\\ &= q(E\hat{\jmath} + vB\ \hat{\imath} \times \hat{k})\\ &= q(E\hat{\jmath} + vB(-\hat{\jmath}))\\ &= q(E - vB)\hat{\jmath}\end{aligned}$$

So we see that the force will always be in the positive or negative y direction with the sign being determined by the sign of $q(E - vB)$.
(a) If $v = E/B$ then $q(E - vB) = 0$, and the Lorentz force is zero.
(b) If $v > E/B$ then $(E - vB) < 0$, and the Lorentz force in the negative direction for a positive particle.
(c) and in the positive direction for negative particles.
(d) If $v < E/B$ then $(E - vB) > 0$, and the Lorentz force in the positive direction for a positive particle.

In words we can just say that when $v = E/B$ the magnetic force and electric force is balanced. Since the magnetic force is proportional to the velocity the magnetic force will be stronger when the velocity is greater than this "balancing" velocity and the electric force will be stronger if the velocity is lower than the "balancing" velocity.

4.7 By the right-hand-rule we have, west, 0, up, down. We can also do this problem more algebraicly. Set our coordinates so that $\hat{\imath}$ is east $\hat{\jmath}$ is north and $\hat{k}$ is upward. In this system $\vec{B} = B\hat{\jmath}$ so that

$$\begin{aligned}\vec{F} &= q\vec{v} \times \vec{B} = -e\left(v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}\right) \times B\hat{\jmath}\\ &= -eB\left(v_x\hat{\imath} \times \hat{\jmath} + v_y\hat{\jmath} \times \hat{\jmath} + v_z\hat{k} \times \hat{\jmath}\right)\\ &= -eB\left(v_x\hat{k} + v_y(0) + v_z(-\hat{\imath})\right) = -eB\left(v_x\hat{k} - v_z\hat{\imath}\right)\end{aligned}$$

Thus for (a) $\vec{v} = -v\hat{k} \longrightarrow v_x = v_y = 0$ and $v_z = -v$ so that $\vec{F} = -eB\left(0\hat{k} - (-v)\hat{\imath}\right) = -evB\hat{\imath}$ and the direction is west.
For (b) $\vec{v} = v\hat{\jmath}$ so $\vec{F} = 0$.
For (c) $\vec{v} = -v\hat{\imath}$ so $\vec{F} = evB\hat{k}$ or up.
For (d) $\vec{v} = \frac{v}{\sqrt{2}}(\hat{\imath} - \hat{\jmath})$ so $\vec{F} = -e\frac{v}{\sqrt{2}}B\hat{k}$.

4.8 The field is in the positive z direction.

4.9 This question is asking us to relate acceleration and force so we must start with Newton's second law.

$$\Sigma\vec{F} = m\vec{a}$$
$$q\vec{E} + q\vec{v}\times\vec{B} = m\vec{a}$$
$$qE\hat{k} + qv\hat{\imath}\times(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}) = ma\hat{k}$$
$$qE\hat{k} + qv(B_x(0) + B_y\hat{k} - B_z\hat{\jmath}) = ma\hat{k}$$
$$(qE + qvB_y - ma)\hat{k} + (-qvB_z)\hat{\jmath} = 0$$
$$qE + qvB_y - ma = 0 \quad \text{and} \quad qvB_z = 0$$
$$B_y = \frac{1}{v}\left(\frac{ma}{q} - E\right) = -2.6\times10^{-3}\text{T} \quad \text{and} \quad B_z = 0$$

Notice that we cannot determine B_x and indeed any value would give the same results. The given measured quantities do not determine the B-field uniquely.

4.10 The magnetic force is due west. The component of the field perpendicular to the velocity is the vertical component of the field which is $B\sin 60°$ so the magnitude of the force is $F = qvB_\perp = qvB\sin 60° = 2.6\times10^{-11}$N

4.11 Plug and chug.

$$\vec{F} = q\vec{v}\times\vec{B} = q\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} = e\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -4 & 1 \\ 1 & 2 & -3 \end{vmatrix}\tfrac{\text{N}}{\text{C}}$$
$$= e(10\hat{\imath} + 7\hat{\jmath} + 8\hat{k})\tfrac{\text{N}}{\text{C}}$$
$$= (1.6\hat{\imath} + 1.1\hat{\jmath} + 1.3\hat{k})\times10^{-18}\text{N}$$
$$F = 2.34\times10^{-18}\text{N}$$

4.12 Looks like we need to go back to the definition of work for this.

$$W = \int\vec{F}\cdot d\vec{s} = \int(q\vec{v}\times\vec{B})\cdot d\vec{s} = \int(q\vec{v}\times\vec{B})\cdot\vec{v}dt = 0$$

Since $(q\vec{v}\times\vec{B})$ must be perpendicular to $\vec{v}$ and thus the dot product is zero.

4.13 Since the field is uniform and the wires straight we can use the relationship $\vec{F} = I\vec{L}\times\vec{B}$ with $\vec{B} = B\hat{\jmath}$.

$$\vec{L}_{ab} = -\ell\hat{\jmath} \text{ so } \vec{F}_{ab} = I(-\ell\hat{\jmath})\times B\hat{\jmath} = -I\ell B\hat{\jmath}\times\hat{\jmath} = 0.$$
$$\vec{L}_{bc} = \ell\hat{k} \text{ so } \vec{F}_{bc} = I(\ell\hat{k})\times B\hat{\jmath} = I\ell B\hat{k}\times\hat{\jmath} = -I\ell B\hat{\imath}.$$
$$\vec{L}_{cd} = -\ell\hat{\imath} + \ell\hat{\jmath} \text{ so } \vec{F}_{cd} = I(-\ell\hat{\imath} + \ell\hat{\jmath})\times B\hat{\jmath} = -I\ell B\hat{k}.$$
$$\vec{L}_{da} = \ell\hat{\imath} - \ell\hat{k} \text{ so } \vec{F}_{da} = I(\ell\hat{\imath} - \ell\hat{k})\times B\hat{\jmath} = I\ell B(\hat{k} + \hat{\imath}).$$

Notice that the sum is zero.

4.14

$$R = \frac{1}{B}\sqrt{\frac{2mV}{e}} \longrightarrow R = 1.98\text{cm}$$

4.15 $\Delta K + \Delta U = W_{\text{nc}}$ so $K_f - 0 + q\Delta V = 0$ thus $v = \sqrt{-2q\Delta V/m} = \sqrt{2qV/m}$. Now that we have the velocity we can find the radius $\Sigma F = ma \longrightarrow qvB = m\frac{v^2}{r} \longrightarrow r = \frac{mv}{qB}$

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}} = 1.98\text{cm}$$

4.16 (a) up, (b) out, (c) none, (d) in.

4.17 $\vec{a} = \Sigma\vec{F}/m = [q\vec{E} + q\vec{v}\times\vec{B}]/m = \frac{q}{m}[\vec{E} + \vec{v}\times\vec{B}]$. So

$$\vec{a} = \frac{q}{m}[50\hat{\jmath} + 200\hat{\imath}\times(0.2\hat{\imath} + 0.3\hat{\jmath} + 0.4\hat{k})]\tfrac{\text{N}}{\text{C}}$$
$$= \frac{q}{m}[50\hat{\jmath} + (0 + 60\hat{k} - 80\hat{\jmath})]\tfrac{\text{N}}{\text{C}} = \frac{q}{m}[60\hat{k} - 30\hat{\jmath}]\tfrac{\text{N}}{\text{C}}$$
$$= 2.87\times10^9(2\hat{k} - \hat{\jmath})\tfrac{\text{m}}{\text{s}^2}$$

5.1 From $t = \pi/2$ to $t = \pi$.

5.2 The peak of the parabola is at the point (0,1). With the parameterization $\vec{r}(t) = \vec{a} + bt\hat{\imath} + ct^2\hat{\jmath}$ the peak occurs when $t = 0$. So that $\vec{a} = 0\hat{\imath} + 1\hat{\jmath}$, and

$$\vec{r}(t) = bt\hat{\imath} + (1 + ct^2)\hat{\jmath}$$

Now we know for some value of t that we will reach the point (1,0) so that

$$\vec{r}(t) = bt\hat{\imath} + (1 + ct^2)\hat{\jmath} = 1\hat{\imath} + 0\hat{\jmath} \longrightarrow \begin{array}{c} bt = 1 \\ 1 + ct^2 = 0 \end{array} \longrightarrow c = -b^2$$

so that

$$\vec{r}(t) = bt\hat{\imath} + (1 - b^2t^2)\hat{\jmath}$$

This already goes through (-1,0) at $t = -1/b$, so that we are done. Note that we can choose the parameter b as we like, so we might as well let $b = 1$ and then

$$\vec{r}(t) = t\hat{\imath} + (1 - t^2)\hat{\jmath}$$

5.3 The parameterization is

$$\vec{r}_s(t) = t\hat{\imath}$$

So

$$\vec{r}_f - \vec{r}_s(t) = -t\hat{\imath} + y\hat{\jmath}$$

$$|\vec{r}_f - \vec{r}_s(t)|^2 = t^2 + y^2$$

$$|\vec{r}_f - \vec{r}_s(t)|^3 = (t^2 + y^2)^{3/2}$$

and

$$\frac{d\vec{r}_s}{dt} = \hat{\imath}$$

and

$$\frac{d\vec{r}_s}{dt} \times [\vec{r}_f - \vec{r}_s(t)] = \hat{\imath} \times (-t\hat{\imath} + y\hat{\jmath}) = y\hat{k}$$

Now putting this into the parameterized form of the Biot-Savart law we find:

$$\vec{B}(\vec{r}_f) = \frac{\mu_0 I}{4\pi} \int \frac{\frac{d\vec{r}_s}{dt} \times [\vec{r}_f - \vec{r}_s(t)]}{|\vec{r}_f - \vec{r}_s(t)|^3} dt = \frac{\mu_0 I}{4\pi} \int_a^b \frac{y\hat{k}}{(t^2 + y^2)^{3/2}} dt$$

$$= \frac{\mu_0 I}{4\pi y} \left[\frac{t}{\sqrt{t^2 + y^2}} \right]_a^b \hat{k}$$

5.4 The parameterization is

$$\vec{r}_s(t) = a\cos t\hat{\imath} + a\sin t\hat{\jmath}$$

where t goes from 0 to θ. The field point is $\vec{r}_f = 0$. So

$$\vec{r}_f - \vec{r}_s(t) = -a\cos t\hat{\imath} - a\sin t\hat{\jmath}$$

$$|\vec{r}_f - \vec{r}_s(t)|^2 = a^2\cos^2 t + a^2\sin^2 t = a^2$$

$$|\vec{r}_f - \vec{r}_s(t)|^3 = a^3$$

and

$$\frac{d\vec{r}_s}{dt} = -a\sin t\hat{\imath} + a\cos t\hat{\jmath}$$

and

$$\frac{d\vec{r}_s}{dt} \times [\vec{r}_f - \vec{r}_s(t)] = (-a\sin t\hat{\imath} + a\cos t\hat{\jmath}) \times (-a\cos t\hat{\imath} - a\sin t\hat{\jmath}) = a^2\hat{k}$$

Now putting this into the parameterized form of the Biot-Savart law we find:

$$\vec{B}(\vec{r}_f) = \frac{\mu_0 I}{4\pi} \int \frac{\frac{d\vec{r}_s}{dt} \times [\vec{r}_f - \vec{r}_s(t)]}{|\vec{r}_f - \vec{r}_s(t)|^3} dt = \frac{\mu_0 I}{4\pi} \int_0^\theta \frac{a^2\hat{k}}{a^3} dt = \frac{\mu_0 I}{4\pi} \frac{\theta}{a} \hat{k}$$

5.5 By employing Ampere's law we find that the field at a radius r is $B(a) = \mu_o I_{\text{in}}/2\pi a$. But for $a < R$

$$I_{\text{in}} = \int J\, dA = \int_0^a J\, 2\pi r dr = \int_0^a \frac{I}{2\pi R r} 2\pi r dr = I\frac{a}{R}$$

Thus

$$B(a) = \frac{\mu_o I_{\text{in}}}{2\pi a} = \frac{\mu_o I \frac{a}{R}}{2\pi a} = \frac{\mu_o I}{2\pi R}$$

For $a > R$ we know that $I_{\text{in}} = I$ so $B = \mu_o I/2\pi a$.

5.6

(a) The force is repulsive.

(b)

5.7 $B = \mu_0 I/2\pi r = 2 \times 10^{-7}$T.

5.8 With $\hat{k}$ along the axis of the loop of radius a the field at the center is

$$\vec{B} = \int d\vec{B} = \frac{\mu_o}{4\pi} \int \frac{Id\vec{s} \times \hat{r}}{r^2} = \frac{\mu_o}{4\pi} \int \frac{Ids\hat{k}}{a^2} = \frac{\mu_o I\hat{k}}{4\pi a^2} \int ds$$

$$= \frac{\mu_o I\hat{k}}{4\pi a^2} 2\pi a = \frac{\mu_o I\hat{k}}{2a} \longrightarrow a = \frac{\mu_o I}{2B} = 31.4\text{cm}$$

5.9 It will take a time of $\Delta t = \Delta x/v = 2\pi r/v = 1.56 \times 10^{-16}$s for the electron to go around the proton and thus electron current will be $I = dq/dt = e/\Delta t =$ 1.0mA. From the previous problem the field at the center of a current loop is $B = \mu_o I/2r = 11.9$T

5.10 Since the point P is along the axis of the horizontal part of the wire this wire will not contribute to the B-field at the point P. So lets just find the field do to the vertical part.

$$\begin{aligned}\vec{B} &= \int d\vec{B} = \frac{\mu_o}{4\pi}\int \frac{Id\vec{s}\times\hat{r}}{r^2} = \frac{\mu_o I}{4\pi}\int \frac{d\vec{s}\times\vec{r}}{r^3}\\ &= \frac{\mu_o I}{4\pi}\int_{-\infty}^{0}\frac{dy\hat{\jmath}\times\vec{r}}{r^3} = \frac{\mu_o I}{4\pi}\int_{-\infty}^{0}\frac{\hat{\jmath}\times(x\hat{\imath}+y\hat{\jmath})}{(x^2+y^2)^{3/2}}dy\\ &= \frac{\mu_o I}{4\pi}\int_{-\infty}^{0}\frac{-x\hat{k}+0}{(x^2+y^2)^{3/2}}dy = -\frac{\mu_o I}{4\pi}x\hat{k}\int_{-\infty}^{0}\frac{dy}{(x^2+y^2)^{3/2}}\\ &= -\frac{\mu_o I}{4\pi}x\hat{k}\int_{-\pi/2}^{0}\frac{x\sec^2\theta d\theta}{(x^2+x^2\tan^2\theta)^{3/2}}\\ &= -\frac{\mu_o I}{4\pi}x\hat{k}\int_{-\frac{\pi}{2}}^{0}\frac{x\sec^2\theta d\theta}{x^3\sec^3\theta} = -\frac{\mu_o I}{4\pi x}\hat{k}\int_{-\frac{\pi}{2}}^{0}\cos\theta d\theta = \frac{\mu_o I}{4\pi x}\hat{k}\end{aligned}$$

5.11 Since $d\vec{s}$ and $\hat{r}$ are parallel for the radial lines, these parts of the wires do not contribute to the B-field at the point of interest. The field contributed by a circular section of radius r and subtended angle θ is found just like in problem 30.3 but this time $\int ds = \Delta s = r\theta$ so the field is $B_{r,\theta} = \frac{\mu_o I}{4\pi r^2}r\theta = \frac{\mu_o I\theta}{4\pi r}$. For the circle at radius b the current is CW so that the field is into the paper at the center while the circle at a is CCW and the field due to it is out of the paper. Choosing out of the paper as $\hat{k}$ we can write

$$\vec{B} = \vec{B}_{a,\theta}\hat{k} + B_{b,\theta}(-\hat{k}) = \frac{\mu_o I\theta}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)\hat{k}$$

Since $b > a$ we know that this will be out of the page.

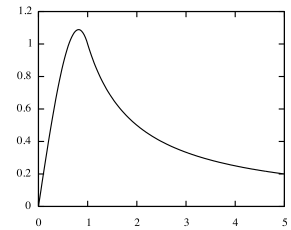
5.12 The magnitude of the B-field at a distance r from a long straight wire is given by $B = \frac{\mu_o I}{2\pi r}$. Thus the force on one wire is due to the field of the other $F = I_1 L B_2 = I_1 L\frac{\mu_o I_2}{2\pi r}$. So we find that $\frac{F}{L} = \frac{\mu_o I_1 I_2}{2\pi r} = 80\mu\text{N}/m$.

5.13 Pick a circular path of radius r around the axis of the wire. We know that the field follows this path and the field strength along this path is a constant, so that $\oint \vec{B}\cdot d\vec{s} = B2\pi r$. But Ampere's law tells us that this must proportional to the current through the closed path. Thus $B2\pi r = \mu_o I_{\text{in}}$ and we can find the field $B = \frac{\mu_o I_{\text{in}}}{2\pi r}$ at a radius r if we can find I_{in}.

$$\begin{aligned}I_{\text{in}} &= \int JdA = \int_0^r J2\pi r dr = \int_0^r J_0\left(1-\left(\frac{r^2}{R^2}\right)\right)2\pi r dr\\ &= 2\pi J_0\left(\tfrac{1}{2}r^2 - \tfrac{1}{4}\left(\frac{r^4}{R^2}\right)\right) \quad \text{for} \quad r < R.\end{aligned}$$

For $r > R$ we find $I_{\text{in}} = 2\pi J_0\left(\frac{1}{2}R^2 - \frac{1}{4}\left(\frac{R^4}{R^2}\right)\right) = 2\pi J_0\frac{1}{4}R^2$. Thus

$$B = \frac{\mu_o I_{\text{in}}}{2\pi r} = \begin{cases}\mu_o J_0\left(\frac{1}{2}r - \frac{1}{4}\left(\frac{r^3}{R^2}\right)\right) & \text{for } r < R\\ \mu_o J_0\frac{1}{4}\frac{R^2}{r} & \text{for } r > R\end{cases}$$



If we let $B_o = \mu_o J_0 R/4$ and $x = r/R$ then this becomes

$$B = \begin{cases}B_0(2x - x^3) & \text{for } x < 1\\ B_0\frac{1}{x} & \text{for } x > 1\end{cases}$$

Graphing B verses x we get the figure to the right. From the figure we can see that the maximum occurs for $x < 1$ and thus we can maximize the function $B_0(2x - x^3)$ to find the maximum field.

$$dB/dx = B_0(2 - 3x^2) = 0 \longrightarrow x = \sqrt{2/3}.$$

Thus $r_{\max} = \sqrt{2/3}R$ and $B_{\max} = \sqrt{32/27}B_0$

5.14 As in previous problems with cylindrical symmetry, the field at a radius r is $B = \mu_o I_{\text{in}}/(2\pi r)$. At $r = a$ the current through the loop is 1.00 A and out of the page so that the field is CW and of magnitude $B = 0.20$mT. At $r = b$ the current through the loop is 2.00 A and into the page so that the field is CCW and of magnitude $B = 0.13$mT.

5.15 Again by employing Ampere's law we find that the field at a radius r is $B = \mu_o I_{\text{in}}/2\pi r$. But for $r_1 < R$

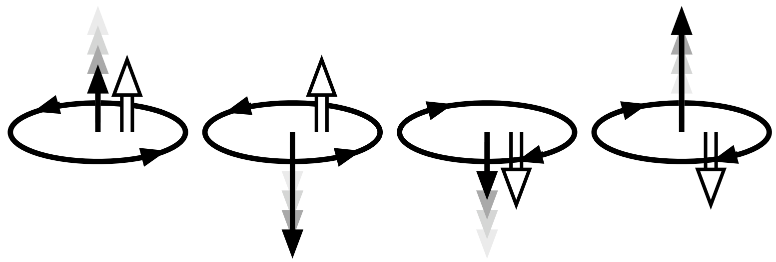
$$I_{\text{in}} = \int JdA = \int_0^r J2\pi r dr = \int_0^r br2\pi r dr = \frac{2\pi}{3}br^3$$

For $r = R$ we get the entire current so $I = I_{\text{in}} = 2\pi bR^3/3$ so that we find $b = 3I/2\pi R^3$. Thus at a general radius $r < R$

$$I_{\text{in}} = I\frac{r^3}{R^3} \longrightarrow B = \frac{\mu_o I_{\text{in}}}{2\pi r} = \frac{\mu_o}{2\pi r}I\frac{r^3}{R^3} = \frac{\mu_o I r^2}{2\pi R^3}$$

For $r > R$ we know that $I_{\text{in}} = I$ so $B = \mu_o I/2\pi r$.

6.1 First redraw the figures with the induced field shown.



In quarters 1 and 3 the fields are in the same directions and the flux is decreasing, thus the induced field is helping out the decreasing flux trying to keep it from decreasing. In quarters 2 and 4 the fields are in opposite directions and the flux is increasing, thus the induced field is trying to stop the flux from increasing.

6.2 We pick our Amperian loop to be a circle with radius r between the plates. Then the magnetic field is parallel to the loop and the electric field is parallel to the normal to the loop. Also there is no current density between the plates so we can simplify Ampere's law as follows.

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \int \vec{J}\cdot d\vec{A} + \mu_0\epsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A}$$

$$B2\pi r = 0 + \mu_0\epsilon_0 \frac{d}{dt}\left[at\pi r^2\right]$$

$$B2\pi r = \mu_0\epsilon_0 a\pi r^2$$

$$B = \tfrac{1}{2}\mu_0\epsilon_0 a r = 1.0\times 10^{-8}\text{T}$$

Notice that the induced magnetic field is very small compared with the electric field that is causing it.

6.3 The loop rule gives us.

$$V_S - \mathcal{E} - \Delta V = 0$$

$$\longrightarrow\ V_S - L\frac{dI}{dt} - IR = 0$$

(a) There is a steady state solution to the above equation: $I_{ss}(t) = V_S/R$. Let us try a solution of the form

$$I(t) = f(t) + V_S/R\ \longrightarrow\ \frac{dI}{dt} = \frac{df}{dt}$$

Substituting this into $V_S - L\frac{dI}{dt} - IR = 0$ we find

$$V_S - L\frac{df}{dt} - \left(f(t) + \frac{V_S}{R}\right)R = 0$$

$$\longrightarrow\ -L\frac{df}{dt} - f(t)R = 0$$

$$\longrightarrow\ \frac{df}{dt} = -\frac{R}{L}f(t)$$

$$\longrightarrow\ f(t) = Ce^{-\frac{R}{L}t}$$

$$\longrightarrow\ I(t) = Ce^{-\frac{R}{L}t} + \frac{V_S}{R}$$

Now we know that at $t=0$ the current is zero so that

$$0 = Ce^0 + \frac{V_S}{R}\ \longrightarrow\ C = -\frac{V_S}{R}$$

$$\longrightarrow\ I(t) = \frac{V_S}{R}\left[1 - e^{-\frac{R}{L}t}\right]$$

(b)

$$\mathcal{E} = L\frac{dI}{dt} = L\frac{V_S}{R}\frac{R}{L}e^{-\frac{R}{L}t} = V_S e^{-\frac{R}{L}t}$$

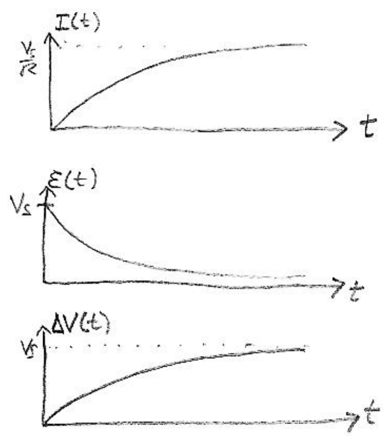
(c)

$$\Delta V = IR = V_S\left[1 - e^{-\frac{R}{L}t}\right]$$

(d) The loop rule says the following sum should be zero.

$$V_s - \mathcal{E} - \Delta V = V_s - V_S e^{-\frac{R}{L}t} - V_S\left[1 - e^{-\frac{R}{L}t}\right] = 0 \qquad \text{OK}$$

(e)



6.4

(a)

$$P_{\text{avg}} = I_0^2 R/2\ \longrightarrow\ I_0^2 R = 2P_{\text{avg}} = 120\text{W}$$

(b) But the supply from the outlet is 120 volts so that

$$I_0 R = V_0 = (120\text{V})\sqrt{2} = 170\text{V}$$

Combining these two equations we can solve for the current:

$$I_0 = \frac{I_0^2 R}{I_0 R} = \frac{120\text{W}}{170\text{V}} = 0.706\text{A}$$

Now we can compute

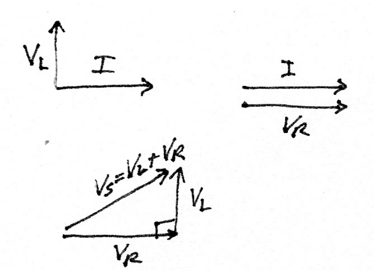
$$R = \frac{I_0 R}{I_0} = \frac{170\text{V}}{0.706\text{A}} = 240\Omega$$

(c) Yes.

6.5 Kirchhoff's loop rule gives us that

$$V_S(t) - V_L(t) - V_R(t) = 0 \longrightarrow V_S(t) = V_L(t) + V_R(t)$$

In the phasor diagram this implies that the sum of the phasors for V_L and V_R must be equal to the phasor of the source voltage V_S. Since the resistor phasor is parallel to the current phasor we know that the phasors for V_L must lead the phasor for V_R by 90°. Thus the sum (also V_S) forms the hypotenuse of a right triangle.



Since the lengths of the phasors are the amplitudes of the voltages we can use the pythagorian theorem to find that

$$V_{S_0}^2 = V_{R_0}^2 + V_{L_0}^2 = (RI_0)^2 + (Z_L I_0)^2$$

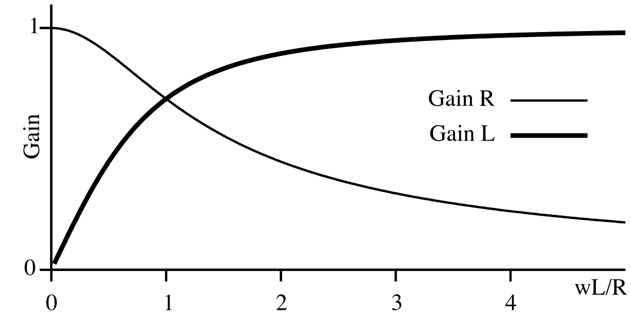
$$\longrightarrow \frac{V_{S_0}^2}{I_0^2} = R^2 + Z_L^2$$

$$\frac{V_{S_0}}{I_0} = \sqrt{R^2 + Z_L^2} = \sqrt{R^2 + (\omega L)^2}$$

Now we can find the gains.

$$G_R(\omega) \equiv \frac{V_{R_0}}{V_{S_0}} = \frac{V_{R_0}/I_0}{V_{S_0}/I_0} = \frac{R}{\sqrt{R^2 + (\omega L)^2}} = \frac{1}{\sqrt{1 + (\omega L/R)^2}}$$

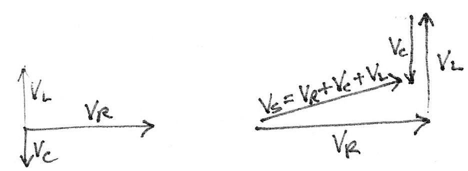
$$G_L(\omega) \equiv \frac{V_{L_0}}{V_{S_0}} = \frac{V_{L_0}/I_0}{V_{S_0}/I_0} = \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}} = \frac{1}{\sqrt{(R/\omega L)^2 + 1}}$$



We see that the resistor has a higher gain for low frequencies.

6.6 From the loop rule we find

$$V_S(t) = V_R(t) + V_L(t) + V_C(t)$$



From the phasor diagram we see that the amplitude of V_S forms the hypotenuse of a right triangle with the other two sides having lengths (V_{R_0}) and $(V_{L_0} - V_{C_0})$. Thus from the pythagorian theorem we find the following.

$$V_{S_0}^2 = V_{R_0}^2 + (V_{L_0} - V_{C_0})^2 = (RI_0)^2 + (Z_L I_0 - Z_C I_0)^2$$

Solving for I_0 we find

$$I_0 = \frac{1}{\sqrt{R^2 + (Z_L - Z_C)^2}} V_{S_0}$$

This will be maximized when the denominator is minimized, and this will occur when $Z_L = Z_C$ that is when ω is such that $\omega L = \frac{1}{\omega C} \longrightarrow \omega = \sqrt{\frac{1}{LC}}$.

6.7 $\Phi_1 = \vec{B} \cdot \vec{A}_1 = -BA_1 \cos\theta$. The total flux $\Phi_1 + \Phi_2 = 0$ since together they form a closed surface and $\oint \vec{B} \cdot d\vec{A} = 0$. Thus $\Phi_2 = -\Phi_1 = BA_1 \cos\theta$.

6.8 $I = \frac{V}{R} = \frac{Nd\Phi/dt}{R} = \frac{NA\Delta B}{R\Delta t} = \frac{200(0.20\text{m}^2)(1.6\text{T})}{R(0.020\text{s})} = 160\text{A}$.

6.9 $\mathcal{E} = -\frac{d}{dt}\Phi = -\frac{d}{dt}[AB_o e^{-t/\tau}] = \frac{AB_o}{\tau} e^{-t/\tau}$.

6.10 Since we know that the field is parallel to the normal of the area at all points we can write $\Phi = \int \vec{B} \cdot d\vec{A} = \int B dA$. But he field strength at a distance r from a current carrying wire is $B = \frac{\mu_o I}{2\pi r}$ so we can write the integral over the area as an integral over r with the area elements $dA = cdr$.

$$\Phi = \int BdA = \int_a^{a+b} \frac{\mu_o I}{2\pi r} cdr = \frac{\mu_o Ic}{2\pi} \ln\left(\frac{a+b}{a}\right)$$

Thus

$$\mathcal{E} = -N\frac{d\Phi}{dt} = -N\frac{\mu_o c}{2\pi} \ln\left(\frac{a+b}{a}\right)\frac{dI}{dt}$$
$$= -N\frac{\mu_o c}{2\pi} \ln\left(\frac{a+b}{a}\right) I_o\omega \cos(\omega t+\phi)$$

6.11 Picking a closed loop of radius r as the path we can use $\oint \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A}$ to find the electric field. Since we know that $\vec{B}$ is perpendicular to the area and $\vec{E}$ is parallel to the perimeter, this integral equation becomes

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A} \longrightarrow E2\pi r = -\frac{d}{dt}BA.$$

But $-\frac{d}{dt}BA = -A\frac{dB}{dt} = -\pi r^2(0.060t\ \text{T})$. So that we find $E = -\frac{1}{2}r(0.060t\ \text{T}) = -0.0018\text{N/C}$. The negative in this case implies CCW since the direction implicitly chosen for the normal to the area was in the direction of B and thus the direction of the path was CW.

6.12 As in the previous problem

$$E2\pi r = -\frac{d}{dt}BA \longrightarrow E = -\frac{1}{2\pi r}A\frac{dB}{dt}.$$

But this time $A\frac{dB}{dt} = \pi R^2(6.0t^2 - 8.0t)\text{T}$. So that

$$E = -\frac{R^2}{r}(3.0t^2 - 4.0t)\text{T} = (-0.375t^2 + 0.5t)\text{N/C}$$

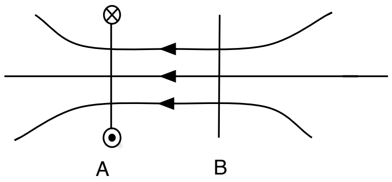
At $t = 2.0$s we find $E = -(0.5)$N/C the negative again indicates a CCW field. The force on an electron will then be $F = qE = eE = 8.0\times10^{-20}$N in the CW direction.

6.13 $Q = \int \frac{dq}{dt}dt = \int Idt = \int \frac{V}{R}dt = \int \frac{N}{R}\frac{d\Phi}{dt}dt = \frac{N}{R}\int d\Phi = \frac{N}{R}\Delta\Phi = \frac{N}{R}\Delta[BA] = \frac{N}{R}\Delta[B]A = \frac{N}{R}[B_f - B_i]A = \frac{200}{5.0\Omega}[2.20\text{T}](0.01\text{m}^2) = 0.88\text{C}$.

6.14 The flux is $\Phi = \int \vec{B}\cdot d\vec{A} = \vec{B}\cdot\vec{A} = BA\cos\omega t$. Thus $\mathcal{E} = -d\Phi/dt = BA\omega\sin\omega t = (3.016\text{V})\sin\omega t$, and the current is $I = \mathcal{E}/R = (BA\omega/R)\sin\omega t = (3.016\text{A})\sin\omega t$. The power dissipated in the loop is $P = IV = (9.096\text{J})\sin^2\omega t$. From the force on a current is $F = I\vec{L}\times\vec{B}$ we find that the torque on a current loop is $\tau = I\vec{A}\times\vec{B} = IAB\sin\omega t = (A^2B^2\omega/R)\sin^2\omega t$

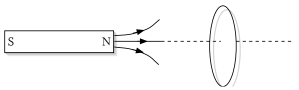
6.15
(a) $\phi = N\mu_0 nIa^2$
(b) $\phi = N\mu_0 nIc^2$

6.16 Look at the loops from the side:

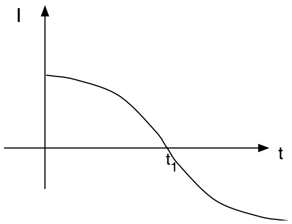


(a) Increasing the current means the magnetic field increases, meaning the magnetic flux through loop B is increasing. Thus, the current induced in B must cause a magnetic field opposite that of the field caused by A. So the current flows clockwise. (b) Since the magnetic field of B is opposite that of A, they act like magnets with like poles facing each other–they repel. If the current is decreasing, then the induced field, and so the induced current flows in the opposite direction as before. In this case the loops will attract.

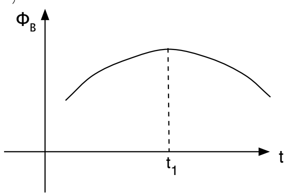
6.17



(a) As the magnet moves toward the loop, the flux through the loop will increase. After the magnet is halfway through the loop, the flux will begin to decrease. A graph of flux versus time will look something like:



(b) As the magnet moves in, the induced current will be counterclockwise since the induced magnetic field will be opposite the magnet's field. Using positive current as counterclockwise, the derivative of the curve above gives:



6.18
(a) After the switch has been closed for a long time, the inductor acts just like a wire, so the current flowing through the 100Ω resistor is zero and the current flowing through 10Ω resistor is $I_0 = 10/10 = 1$A. This is also the current flowing through the inductor.

(b) Open the switch, at that instant there is a current flowing through the inductor which begins to immediately flow through the 100 Ω resistor. The initial current through the 100 Ω resistor is 1A, so the initial potential difference is $V_R = V_L = (1\text{A})(100\Omega) = 100\text{V}$.
(c) The current falls exponentially:

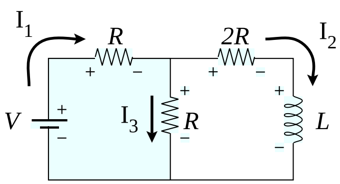
$$I(t) = I_0 e^{-t(100\Omega/2\text{H})} = (1\text{A})e^{-50t}.$$

6.19 $\mathcal{E} = LdI/dt = L\Delta I/\Delta t = 100\text{V}$.

6.20 For a solenoid $L = \mu_o N^2 A/\ell = 13.6\text{mH}$.

6.21 $\mathcal{E} = LdI/dt = LI_{\max}\omega\cos\omega t = (18.8\text{V})\cos\omega t$.

6.22



Let us write out Kirchhoff's rules.

$$I_1 = I_2 + I_3$$
$$V - I_1 R - I_3 R = 0$$
$$I_3 R - I_2 2R - L\frac{dI_2}{dt} = 0$$

We can rewrite the first two equation to give us $I_1 = \frac{1}{2}(\frac{V}{R} + I_2)$ and $I_3 = \frac{1}{2}(\frac{V}{R} - I_2)$ Putting this into the third equation gives us

$$\tfrac{1}{2}(\frac{V}{R} - I_2)R - I_2 2R - L\frac{dI_2}{dt} = 0$$
$$V - 5I_2 R - 2L\frac{dI_2}{dt} = 0$$

Which can be solved by separation of variables. With the initial condition $I = 0$ at $t = 0$ the solution is

$$I_2 = \frac{V}{5R}(1 - e^{-5Rt/2L}) = (0.5\text{A})(1 - e^{-t/\tau}) \quad \text{with} \quad \tau = 0.1\text{s}.$$

This then gives us
$I_1 = \frac{1}{2}\left(\frac{V}{R} + I_2\right) = \frac{1}{2}\left(\frac{V}{R} + \frac{V}{5R}(1 - e^{-t/\tau})\right) = \frac{V}{10R}(6 - e^{-t/\tau})$.

6.23 Taking the derivative we find that

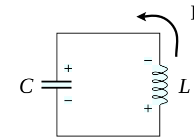
$$dI/dt = I_o e^{-t/\tau}(-1/\tau) = I(-R/L).$$

Plugging this in we find that

$$LdI/dt = -IR \quad \text{and} \quad IR + LdI/dt = 0.$$

6.24 The energy is $U_B = \frac{1}{2}LI^2 = \frac{1}{2}\mu_o N^2 AI^2/\ell = 2.4\mu\text{J}$.

6.25 The capacitor initially be charged to a potential $\mathcal{E}$ and will thus have an initial (and max) charge of $q_0 = C\mathcal{E}$.



After the switch is thrown the battery and resistor are effectively removed from the circuit and thus we just have an LC circuit with an initial charge of $q_0 = C\mathcal{E}$.

$$V_C + V_L = 0 \longrightarrow \frac{q}{C} + L\frac{dI}{dt} = 0 \longrightarrow \frac{d^2q}{dt^2} = -\frac{q}{LC}$$

Which has the solution $q = q_o\cos\omega t$ with $\omega = 1/\sqrt{LC}$. This leads us to the current $I = dq/dt = -q_o\omega\sin\omega t$. The total energy in this ideal circuit is a constant so we can just find the initial energy which is the energy stored on the capacitor $U = \frac{1}{2}QV = \frac{1}{2}q_o\mathcal{E} = \frac{1}{2}C\mathcal{E}^2$.

6.26 Over the time interval $(0, T)$ (where T is the period) the voltage can be written as $V = at + b = \frac{2V_{\max}}{T}t - V_{\max}$ thus
$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T V^2 dt} = \sqrt{\frac{V_{\max}^2}{T}\int_0^T(\frac{4}{T^2}t^2 - \frac{4}{T}t + 1)dt} = \frac{V_{\max}}{\sqrt{3}}$.

6.27 $P_{\text{ave}} = \frac{1}{T}\int_0^T Pdt = \frac{1}{T}\int_0^T \frac{V^2}{R}dt = \frac{1}{T}\int_0^T \frac{V_{\max}^2\sin^2\omega t}{R}dt$ so that $P_{\text{ave}} = \frac{V_{\max}^2}{2R} \longrightarrow R = \frac{V_{\max}^2}{2P_{\text{ave}}}$. Thus we find that $R_{75\text{W}} = 192\Omega$ and $R_{100\text{W}} = 145\Omega$.

6.28 $I = I_{\max}\sin\omega t \longrightarrow V_L = LdI/dt = LI_{\max}\omega\cos\omega t$, and we see that $V_{\max} = LI_{\max}\omega$. Using this we can find $L = V_{\max}/I_{\max}\omega = 42\text{mH}$. Also $\omega = V_{\max}/LI_{\max} = 942\frac{\text{rad}}{\text{s}}$

6.29 If the capacitor is to begin uncharged then the voltage must begin at zero so $V = V_o\sin\omega t$. But

$$q = CV = CV_o\sin\omega t$$

so that

$$I = \frac{dq}{dt} = CV_o\omega\cos\omega t = C\sqrt{2}V_{\text{rms}}\omega\cos\omega t$$

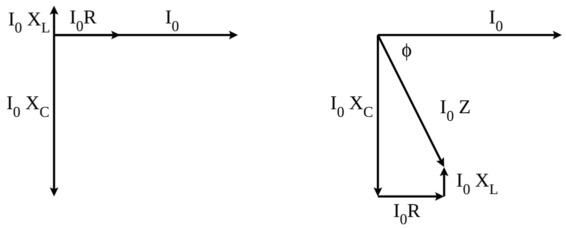
and thus $I(1/180\text{s}) = -32\text{A}$.

6.30 $X_C = 1/\omega C$. Thus if $X_C < 175\Omega$ then $\omega > 1/(C175\Omega) = 260\frac{\text{rad}}{\text{s}} \longrightarrow f > 41\text{Hz}$. The impedance for a capacitor that is twice as big will be half so this is $X_C < 88\Omega$.

6.31 $q = CV = CV_{\max}\cos\omega t$, while $I = dq/dt = -CV_{\max}\omega\sin\omega t$.

6.32 $X_C = X_L \longrightarrow 1/\omega C = \omega L \longrightarrow \omega = 1/\sqrt{LC} = 17.5\times10^3\frac{\text{rad}}{\text{s}} \longrightarrow f = 2.79\text{kHz}$.

6.33 The inductive impedance is $X_L = \omega L = 78.5\Omega$. The capacitive impedance is $X_C = 1/\omega C = 1.59\text{k}\Omega$.



By the figure we see that

$$Z = \frac{I_0 Z}{I_0} = \sqrt{R^2 + (X_L - X_C)^2} = 1.52\text{k}\Omega$$

and that

$$\tan\phi = (X_L - X_C)/R \;\longrightarrow\; \phi = -84.3°.$$

The maximum current is $I_{\max} = V_{\max}/Z = 138\text{mA}$. This current will lead the voltage by 84.3°.

6.34 As in the previous problem,

$$\tan\phi = (X_L - X_C)/R \;\longrightarrow\; \phi = 17.4°.$$

Which implies that the voltage reaches a maximum 17.4° before the current.

6.35 Since $Z = \sqrt{R^2 + (X_L - X_C)^2}$ the maximum current (smallest Z) will be when $X_C = X_L \;\longrightarrow\; \omega_{\text{res}} = \sqrt{\frac{1}{LC}} = 996\frac{\text{rad}}{\text{s}}$. Thus $f = 159\text{Hz}$.

6.36 At $f = 99.7\text{MHz}$ we need $X_C = X_L \;\longrightarrow\; C = 1/\omega^2 L = 1.82\text{pF}$.

7.1

$$\sqrt{\frac{1}{\mu_0\epsilon_0}} = \sqrt{\frac{1}{(4\pi\times10^{-7}\mathrm{N/A^2})(8.854187817\times10^{-12}\mathrm{C^2/N\cdot m^2})}}$$
$$= 2.99792458\times10^8\sqrt{\frac{\mathrm{A^2\cdot m^2}}{\mathrm{C^2}}}$$
$$= 2.99792458\times10^8\sqrt{\frac{\mathrm{m^2}}{\mathrm{s^2}}} = 2.99792458\times10^8\,\frac{\mathrm{m}}{\mathrm{s}}$$

7.2 $x = A\cos\phi \longrightarrow \phi = \cos^{-1}[x/A] = \cos^{-1}[1.4/3.2] = \cos^{-1}[0.4375]$ So $\phi = \pm1.12\mathrm{rad} = \pm64.06°$

7.3
(a) $\phi = 0$.
(b) $\phi = \pi$.
(c) $\phi = \pi/2$.
(d) $\phi = 3\pi/2$.
(e) $v = dx/dt = -A\omega\sin(\omega t+\phi_0)$. So

$$\frac{v_0}{x_0} = \frac{-A\omega\sin(0+\phi_0)}{A\cos(0+\phi_0)} = -\omega\tan\phi_0$$

So

$$\phi_0 = \tan^{-1}\frac{-v_0}{x_0\omega} = \tan^{-1}(-0.3) = -0.291\text{ rad}$$

Note that the other possible angle with the same tangent would be in the second quadrant, but since our position and velocity are both positive we know that our angle must be in the fourth quadrant.

7.4 The oscillator goes through a complete cycle when the phase increases by 2π. Thus

$$\Delta\phi = 2\pi \longrightarrow \omega T = 2\pi \longrightarrow \omega = \frac{2\pi}{T} = 251.3\,\frac{\mathrm{rev}}{\mathrm{s}}$$

7.5 $A = 2$, $\lambda = 3\mathrm{m}$, $T = 6\mathrm{s}$, $f = \frac{1}{6}\mathrm{Hz}$, $\omega = \frac{\pi}{3}\frac{\mathrm{rad}}{\mathrm{s}}$, $k = \frac{2\pi}{3}\mathrm{m}^{-1}$, $v = \frac{1}{2}\frac{\mathrm{m}}{\mathrm{s}}$, $y(x,t) = 2\cos(\frac{2\pi}{6\mathrm{s}}t - \frac{2\pi}{3\mathrm{m}}x) = 2\cos(1.0472\ t - 2.0944\ x)$.

7.6 $\lambda f = c \longrightarrow \lambda = c/f = 0.10\mathrm{m}$.

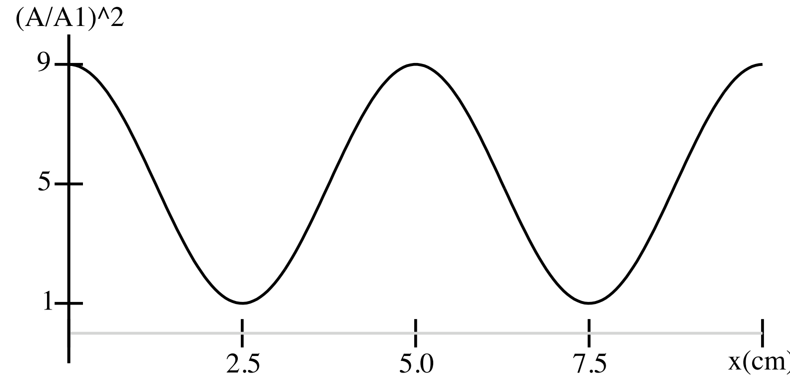
7.7 $\lambda f = c \longrightarrow \lambda = c/f = 3.19\mathrm{m}$.

7.8

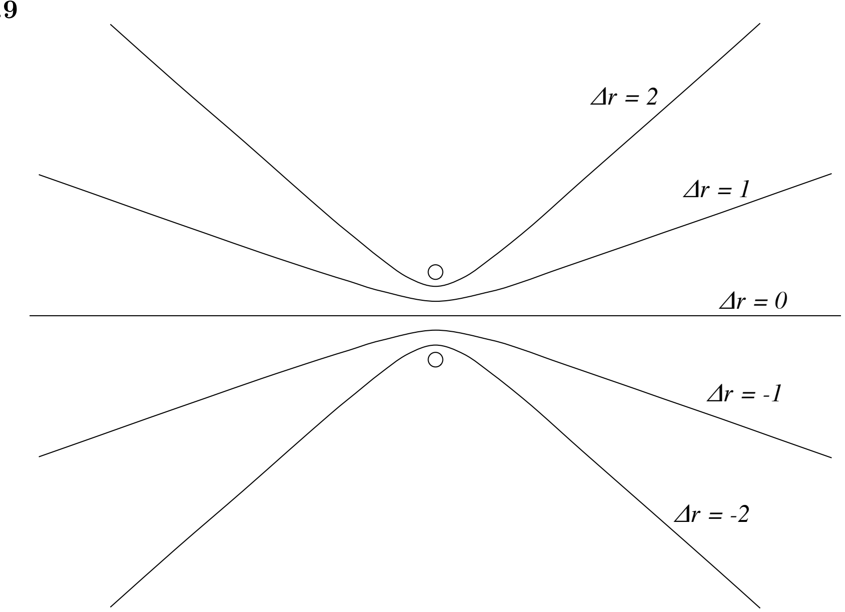
$$\Delta\phi = \phi_2 - \phi_1 = -kr_2 + kr_1 = k(r_1 - r_2) = k\ 2x = \frac{2\pi}{\lambda}2x$$

Also $A_2 = 2A_1$ so that

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_2-\phi_1)$$
$$= A_1^2 + (2A_1)^2 + 2A_1(2A_1)\cos(\frac{2\pi}{\lambda}2x)$$
$$= A_1^2\left[5 + 4\cos(\frac{2\pi}{\lambda}2x)\right]$$



7.9



The maximum and minimum occur when Δr is an even and odd multiples of $\lambda/2$. So when the wavelength is 1cm the even multiples of 0.5cm are the lines we drew. So we drew the maximums for a wavelength of 1cm. The minimums would be half way between the lines we drew. If the wavlength is 2cm then half the wavelength is 1cm and the maximum will occur at the lines with $\Delta r = 0, \pm2$ while the minimums will be a the lines with $\Delta r = \pm1$.

7.10 The maximum at 40 cm must be the maximum that corresponds to the first even multiple, since it is the first maximum from the central maximum (the zeroth multiple). Thus

$$d\sin\theta = 2\frac{\lambda}{2} = \lambda$$

But we can also tell by the geometry of the setup that

$$\tan\theta = \frac{40\text{cm}}{300\text{cm}} = \frac{4}{30}$$

Combining these we find that

$$\lambda = d\sin\theta = (10\text{cm})\sin\left(\tan^{-1}(4/30)\right) = 1.32\text{cm}$$

7.11 The reflection from the front face of the bubble is air-water thus the index of refraction change is low-high and there will be a reflection phase shift. The front path has a phase of $\phi_A = -kr + \pi$. The reflection from the back face of the bubble is water-air or high-low, so there is no reflection phase shift on this path: $\phi_B = -kr$. The phase difference between the paths is then $\phi_A - \phi_B = k\Delta r + \pi = \frac{2\pi}{\lambda'}\Delta r + \pi = \frac{2\pi}{\lambda'}2t + \pi$. Now we see that if $t \ll \lambda$ then $\frac{2\pi}{\lambda'}2t \approx 0$ and $\phi_A - \phi_B \approx \pi$. So as the film gets thin the reflection becomes a minimum not a maximum.

7.12 The path difference is $\Delta = \sqrt{d^2 + 1^2} - 1$, where d is measured in meters. For the minimum d, $\Delta = \lambda/2$. So,

$$\sqrt{d^2 + 1^2} - 1 = \frac{\lambda}{2} \longrightarrow d^2 = \left(1 + \frac{\lambda}{2}\right)^2 - 1 = .103.$$

$$\longrightarrow d = .32\text{m}.$$

7.13 The reflections are both the same so there is no phase difference due to the reflections and

$$\Delta\phi = \Delta\phi_{\text{path}} + \Delta\phi_{\text{reflection}} = 2\pi\frac{\Delta L}{\lambda'} + 0.$$

Since the reflections are a minimum we know that this phase difference is an odd multiple π. But since we want the minimum thickness this will be the first odd multiple (1) and we find that

$$2\pi\frac{\Delta L}{\lambda'} = (1)\pi.$$

Also we know that the path difference is twice the thickness of the film so that $\Delta L = 2t$ and we find using our condition above that

$$t = \frac{\lambda'}{4} = \frac{\lambda/n}{4} = 96\text{nm}.$$

7.14 This time the reflections are not the same and so we do have a half cycle phase difference due to the reflection

$$\Delta\phi = 2\pi\frac{\Delta L}{\lambda'} + \pi$$

The reflections are strong so this must be constructive interference and the phase difference is an even multiple of π:

$$\Delta\phi = 2\pi\frac{\Delta L}{\lambda'} + \pi = m\pi \qquad \text{for } m \text{ even}$$

Again $\Delta L = 2t$ so that

$$2\pi\frac{2t}{\lambda'} + \pi = m\pi \longrightarrow 4t = (m-1)\lambda' \qquad \text{for } m \text{ even}$$

But if m is even then $k = m - 1$ is odd so

$$4t = k\lambda' \qquad \text{for } k \text{ odd}$$

Now the film is reflective for both red and green so

$$4t = k_{\text{red}}\lambda'_{\text{red}} \qquad \text{and} \qquad 4t = k_{\text{green}}\lambda'_{\text{green}}$$

But t is the same for both (there is only one film). Thus

$$k_{\text{green}}\lambda'_{\text{green}} = k_{\text{red}}\lambda'_{\text{red}} \longrightarrow \frac{k_{\text{red}}}{k_{\text{green}}} = \frac{\lambda'_{\text{green}}}{\lambda'_{\text{red}}} = \frac{5}{7}$$

The lowest odd integer k's that will give this ratio are $k_{\text{red}} = 5$ and $k_{\text{green}} = 7$. Thus

$$t = k_{\text{red}}\lambda'_{\text{red}}/4 = 5\lambda'_{\text{red}}/4 = 5\lambda_{\text{red}}/4n = 658\text{nm}$$

7.15 One path has no reflections so there is no phase shift of this path due to reflections. The other path has two hard reflections and thus has two half cycle phase shifts or a total of a full cycle phase shift due to reflections. Thus there is effectively no phase difference (one cycle) due to the difference in reflection of the two paths, and the phase difference is due totally to the path difference ($\Delta L = 2d$). Since we are looking for the first constructive interference we know that

$$2\pi\frac{\Delta L}{\lambda} = \Delta\phi = 2\pi \longrightarrow d = \lambda/2 = 280\text{nm}$$

7.16 We know that the path difference at an angle θ from the central maximum is given by

$$\Delta L = d\sin\theta$$

where d is the slit spacing. Also we know that the phase difference caused by this path difference is

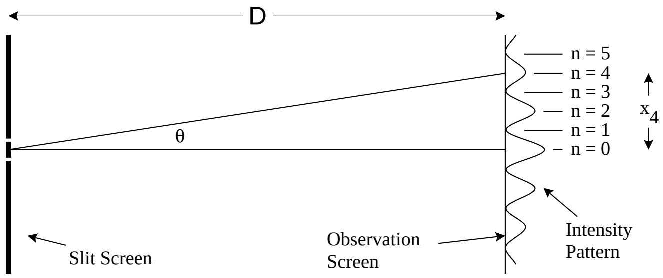
$$\Delta\phi = 2\pi\frac{\Delta L}{\lambda}$$

and that for interference maximums and minimums

$$\Delta\phi = n\pi$$

where n is an integer. Plugging the first and the third equations into the second we get

$$n\pi = 2\pi\frac{d\sin\theta}{\lambda} \longrightarrow d\sin\theta_n = n\frac{\lambda}{2}$$



From the geometry of the situation we see that

$$\sin\theta_n = \frac{x_n}{D}$$

where x_n is the position of the nth extreme. Substituting this into our previous expression we find

$$d\frac{x_n}{D} = n\frac{\lambda}{2} \longrightarrow x_n = n\frac{\lambda D}{2d} = n(1.31\text{mm})$$

Thus $x_2 - x_0 = 2.62$mm and $x_3 - x_1 = 2.62$mm.

7.17 As in the last problem we find

$$d\sin\theta_n = n\frac{\lambda}{2} \longrightarrow \sin\theta_n = n\frac{\lambda}{2d}$$

We are given the speed and frequency so that we can find the wavelength $\lambda = vT = v/f = 0.177$m. With this wavelength we can find the angle of the first maximum.

$$\sin\theta_2 = 2\frac{\lambda}{2d} \longrightarrow \theta_2 = 36.1°$$

For the microwave:

$$d = \frac{\lambda}{\sin\theta} = 5.1\text{cm}$$

For $d = 1\mu$m:

$$\lambda = d\sin\theta = 590 \longrightarrow f = \frac{c}{f} = 5.1\times 10^14\text{Hz}$$

7.18 Use the trig identity:

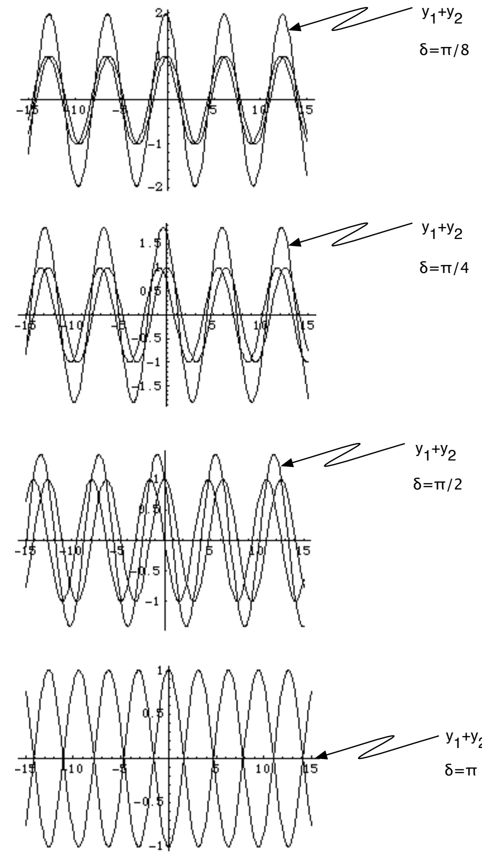
$$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta).$$

$$\alpha+\beta = \omega t - kd_1 + \delta + \omega t - kd_2 = 2\omega t - k(d_1+d_2) + \delta$$

$$\alpha-\beta = \omega t - kd_1 + \delta - (\omega t - kd_2) = k(d_2-d_1) + \delta$$

$$\longrightarrow y_1 + y_2 = 2A\cos\frac{1}{2}(2\omega t - k(d_1+d_2)+\delta)\cos\frac{1}{2}(k(d_2-d_1)+\delta)$$

$$= 2A\cos\left(\omega t - \frac{k}{2}(d_1+d_2) + \frac{\delta}{2}\right)\cos\left(\frac{k}{2}(d_2-d_1)+\delta\right)$$



7.19 There is a path difference in getting from the source to the slits $d\sin\theta_1$ and a path difference in going from the slits to the observation point so that the total path difference is the sum of the two. We must be careful though since the top ray goes farther on the first leg and shorter on the second leg so that

$$\Delta L = (d\sin\theta_1) + (-d\sin\theta_2).$$

So we find the max and mins for even and odds as always: use

$$\Delta\phi = 2\pi\frac{\Delta L}{\lambda} \quad \text{and} \quad \Delta\phi = n\pi$$

to find

$$\Delta L = n\frac{\lambda}{2}$$

We will have an interference maximum when n is even. So if $n = 2m$ with m

an integer we will have an interference maximum and

$$\Delta L = 2m\frac{\lambda}{2} = m\lambda \longrightarrow d(\sin\theta_1 - \sin\theta_2) = m\lambda$$

Thus

$$\sin\theta_1 - \sin\theta_2 = m\lambda/d$$

7.20 The reflections are both the same so there is no phase difference due to the reflections and

$$\Delta\phi = \Delta\phi_{path} + \Delta\phi_{reflection} = 2\pi\frac{\Delta L}{\lambda'} + 0.$$

Since the reflections are a minimum we know that this phase difference is an odd multiple π. But since we want the minimum thickness this will be the first odd multiple (1) and we find that

$$2\pi\frac{\Delta L}{\lambda'} = (1)\pi.$$

Also we know that the path difference is twice the thickness of the film so that $\Delta L = 2t$ and we find using our condition above that

$$t = \frac{\lambda'}{4} = \frac{\lambda/n}{4} = 96\text{nm}.$$

7.21 This time the reflections are not the same and so we do have a half cycle phase difference due to the reflection

$$\Delta\phi = 2\pi\frac{\Delta L}{\lambda'} + \pi$$

The reflections are strong so this must be constructive interference and the phase difference is an even multiple of π:

$$\Delta\phi = 2\pi\frac{\Delta L}{\lambda'} + \pi = m\pi \qquad \text{for } m \text{ even}$$

Again $\Delta L = 2t$ so that

$$2\pi\frac{2t}{\lambda'} + \pi = m\pi \longrightarrow 4t = (m-1)\lambda' \qquad \text{for } m \text{ even}$$

But if m is even then $k = m - 1$ is odd so

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Now the film is reflective for both red and green so

$$4t = k_{\text{red}}\lambda'_{\text{red}} \qquad \text{and} \qquad 4t = k_{\text{green}}\lambda'_{\text{green}}$$

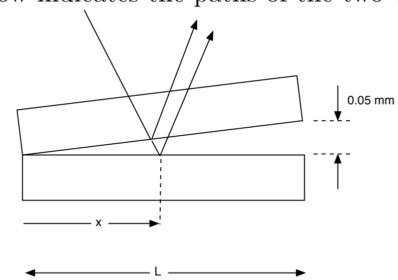
But t is the same for both (there is only one film). Thus

$$k_{\text{green}}\lambda'_{\text{green}} = k_{\text{red}}\lambda'_{\text{red}} \longrightarrow \frac{k_{\text{red}}}{k_{\text{green}}} = \frac{\lambda'_{\text{green}}}{\lambda'_{\text{red}}} = \frac{5}{7}$$

The lowest odd integer k's that will give this ratio are $k_{\text{red}} = 5$ and $k_{\text{green}} = 7$. Thus

$$t = k_{\text{red}}\lambda'_{\text{red}}/4 = 5\lambda'_{\text{red}}/4 = 5\lambda_{\text{red}}/4n = 658\text{nm}$$

7.22 The figure below indicates the paths of the two waves.



The wave that reflects at the glass-air interface has no phase shift due to reflection. The air-glass reflection gains a phase π due to reflection. The path difference between the two waves is 2 times the width of the gap (for near normal incidence). The width of the gap varies as the distance form the end. If x is the distance for the left end (where the plates meet), then the width of the gap is

$$y = x\frac{t}{L},$$

where $t = .05$ mm is the gap at the right end. Thus, the total phase difference between the two reflected rays is:

$$\Delta\phi = \pi + \frac{2\pi}{\lambda}(2y) = \pi + \frac{2\pi}{\lambda}(2x)\frac{t}{L}.$$

For a bright fringe the phase difference must be a multiple of 2π, so

$$2\pi m = \pi + \frac{2\pi}{\lambda}(2y) = \pi + \frac{2\pi}{\lambda}(2x)\frac{t}{L}.$$

$$\longrightarrow \frac{x}{L} = m\frac{\lambda}{2t} \longrightarrow \frac{\Delta x}{L} = \frac{\lambda}{2t},$$

where Δx is the distance between fringes. The number of fringes along the entire length will be:

$$\frac{L}{\Delta x} = \frac{2t}{\lambda} = 166.7 \longrightarrow 166.$$

7.23 $d\sin\theta = m\lambda$ determines the angles where the *bright* maxima occur for two slits separated by a distance d, while $a\sin\theta = m\lambda$ determines the angles where the *dark* minima occur for a single slit of width a.

7.24 This first minimum occurs when $a\sin\theta = \lambda$. There is no angle that satisfies this if $\lambda > a$ since $\sin\theta$ is always less then one. Thus if $a < 637.8$nm there are no diffraction minima.

7.25 As in ID.4 we find the positions (x_n) on the screen where the phase difference is $n\pi$ between the two sides of the slit, to be

$$x_n = n\frac{\lambda D}{2a}$$

where a is the slit width. The difference is that now, (for a single slit), the minima occur when this phase difference is an even multiples of π. The first minimum occurs for $n = 2$ and the third minimum at $n = 6$ thus

$$(3.0\text{mm}) = x_6 - x_2 = 6\frac{\lambda D}{2a} - 2\frac{\lambda D}{2a} = 4\frac{\lambda D}{2a} \longrightarrow a = 230\mu\text{m}$$

7.26 $d \sin\theta = \lambda$ for the first order principal maximum. Compute θ using tangent:

$$\theta = \tan^{-1}\left(\frac{.488}{1.72}\right) = 15.8°.$$

Using $d = 1/5310 = 1.88 \times 10^{-6}$ m:

$$\lambda = (1.88 \times 10^{-6}) \sin(15.8°) = 512\text{nm}$$

7.27 When the lanterns subtend the minimum angle θ_{lanterns} at the observer the images on the retina are just resolved. This means that the central maximum of one image falls on the first minima of the other. For a circular aperture of diameter D the first diffraction minimum occurs when $D \sin\theta_{\text{min}} = 1.22\lambda$ where θ_{min} is the angle between the central maximum and the first minimum. The angle between the images is the same as the angle between the objects (think of the rays going through the center of the lens) so that $\theta_{\text{lanterns}} = \theta_{\text{min}}$. If we let x be the distance between the lanterns and L the distance to the lanterns we find that $\sin\theta_{\text{min}} = x/L$ and thus that $xD/L = 1.22\lambda \longrightarrow x = 50\text{cm}$.

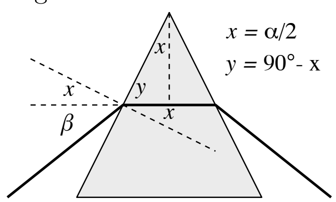
7.28 It is obvious that the answer is 47.4429 since

$$x = \sqrt{\frac{\int_{-\infty}^{\infty} f(t,t') e^{\sin \omega t'} dt'}{1 + \frac{1}{1+\frac{1}{1+\frac{1}{t}}}}}$$

8.1 Let us assume that the person is about $d = 1.7\text{m}$ tall. The sun has a diameter of $D = 1.4 \times 10^6\text{km}$ and is at a distance of $R = 1.5 \times 10^8\text{km}$. Note that if the person was about three times larger she would be about the same size as the sun. Thus if we let r be the distance between the person and the camera then by similar triangles we have that

$$\frac{3d}{r} = \frac{D}{R} \longrightarrow r = 3d\frac{R}{D} = 546\text{m}$$

8.2 By complementary angles we find that.



Now using Snell's law we find

$$\sin(\beta + \frac{\alpha}{2}) = n \sin\frac{\alpha}{s}$$

and thus

$$2\beta = 2\arcsin(n\sin\frac{\alpha}{2}) - \alpha$$

8.3 To Be Done

8.4 To Be Done

8.5 To Be Done

8.6 To Be Done

8.7 To Be Done

8.8 First note that for a diverging lens $f < 0$ so $\frac{1}{f} < 0$ also. Now use the thin lens equation:

$$\begin{aligned} &\frac{1}{x_o} + \frac{1}{x_i} = \frac{1}{f} \\ \longrightarrow\ &\frac{1}{x_o} = \frac{1}{f} - \frac{1}{x_i} \\ \longrightarrow\ &\frac{1}{x_o} < \frac{1}{f} \qquad \text{since } x_i > 0 \\ \longrightarrow\ &\frac{1}{x_o} < 0 \qquad \text{since } \frac{1}{f} < 0 \\ \longrightarrow\ &x_o < 0 \end{aligned}$$

8.9

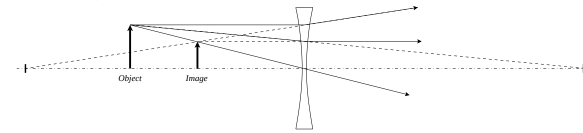
(a) From the magnification equation we find that

$$x_o = -\frac{y_o}{y_i}x_i = -\frac{-8\text{mm}}{2.0\text{m}}(10\text{m}) = 40\text{mm}$$

(b) Now that we have the object distance we can find the focal length from the thins lens equation.

$$f = \left(\frac{1}{x_o} + \frac{1}{x_i}\right)^{-1} = \left(\frac{1}{40\text{mm}} + \frac{1}{10,000\text{mm}}\right)^{-1} = 39.8\text{mm}$$

8.10 Once again we invoke the thin lens and magnification equations to find $x_i = -12.3\text{cm}$ and $y_o/y_i = 0.615$.



8.11 For $p = 40$ and $f = +10$, $q = 13.33$. So, $M = -1/3$. Thus, the image is located 13.33 cm behind the lens, it is inverted, and smaller.

8.12

(a) The ray diagram is sketched below. Here are calculations:

$$\frac{1}{q_1} + \frac{1}{p_1} = \frac{1}{f} \longrightarrow \frac{1}{q_1} + \frac{1}{20} = \frac{1}{10} \longrightarrow q_1 = 20\text{cm}.$$

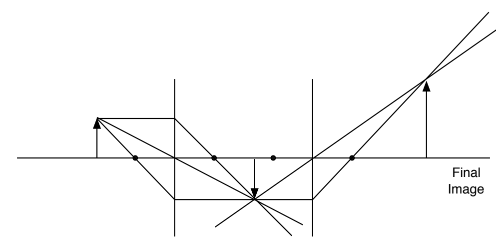
The first image is 20 cm to the right of the first lens, which means it is an object +15 from the second lens:

$$\frac{1}{q_2} = \frac{1}{10} - \frac{1}{15} \longrightarrow q_2 = +30\text{cm}.$$

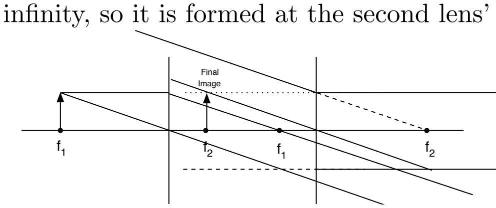
(b) The final image is real and upright.

(c) The magnification is compound:

$$M = m_1 \cdot m_2 = \frac{-q_1}{p_1}\frac{-q_2}{p_2} = +2.$$



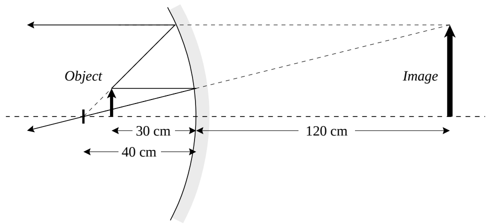
8.13 The ray diagram is below. Since the object is at the first lens' focal point, the image is at infinity. Since the image is at infinity, the object for the second lens is at infinity, so it is formed at the second lens' focal point.



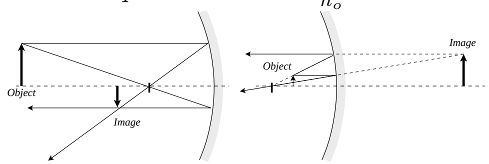
8.14 Since the image is upright $h_i > 0$, and we are given $h_i = 4h_o$. But $\frac{q}{p} = -\frac{h_i}{h_o} = -4 \longrightarrow q = -4p$. We see that p is negative which is as we expect since the image is upright and thus virtual (virtual is when the image is where the light does not actually go, this is on the far side of a mirror). Using the thin lens equation $\frac{1}{q} + \frac{1}{p} = \frac{1}{f}$ we find

$$\frac{1}{-4p} + \frac{1}{p} = \frac{1}{f} \longrightarrow p = \frac{3}{4}f = 30\text{cm} \longrightarrow q = -120\text{cm}$$

Let us check this with the ray diagram.



8.15 Use the thin lens equation $\frac{1}{q} + \frac{1}{p} = \frac{1}{f}$ to find the image position $q = \frac{pf}{p-f}$. Thus since $f = 30\text{cm}$ for $p = 90\text{cm}$ we have $q = 45\text{cm}$ with $\frac{h_i}{h_o} = -\frac{q}{p} = -\frac{1}{2}$, while for $p = 20\text{cm}$ we have $q = -60\text{cm}$ with $\frac{h_i}{h_o} = 3$.



8.16 Here's what you should figure out using $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, with $f = R/2$, and $M = \frac{-q}{p}$:

- $p < f$ virtual, erect image with $M > 1$
- $f < p < 2f$ real, inverted image with $M > 1$
- $p > 2f$ real, inverted image with $M < 1$

8.17 (b) The mirror must be concave, since a convex mirror will always produce a diminished, virtual image. (a) Using the magnification as 5.5:

$$M = -\frac{q}{p} = 5.5,$$

with $p = 2.1$. Thus,

$$q = -Mp = (5.5)(2.1) = -11.5\text{cm}.$$

Find f:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \longrightarrow f = 2.57 \longrightarrow R = 2f = 5.13\text{cm}.$$