1.1 The area between the two paths is $A = \frac{1}{2}Ld$, thus the magnetic flux is $\phi = \frac{1}{2}LdB$ and the phase shift due to the field will be $\Delta\phi = 2\pi\frac{e\frac{1}{2}LdB}{h}$ This phase difference must be π when the first minimum is reached. Thus $2\pi\frac{e\frac{1}{2}LdB}{h} = \pi \longrightarrow B = \frac{h}{eLd}$

2.1

$$\begin{aligned}
\langle K \rangle &= \frac{3}{2} kT \\
\langle \tfrac{1}{2} m v^2 \rangle &= \frac{3}{2} kT \\
\langle v^2 \rangle &= \frac{3kT}{m} \\
v_{\text{rms}} = \sqrt{\langle v^2 \rangle} &= \sqrt{\frac{3kT}{m}}
\end{aligned}$$

Water is 2 hydrogens (2u) and one oxygen (16u) so the mass is 18 grams per mole. $m = \frac{0.018\text{kg}}{N_A}$. Thus $v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kN_AT}{0.018\text{kg}}} = 645\,\frac{\text{m}}{\text{s}}$.

2.2 Let L be the length of the pipe, T_w the temperature of the water and T_a the temperature of the surrounding air. Let k be the thermal conductivity of the insulation and d be the thickness of the insulating layer and D the diameter of the pipe. We can then compute

$$\begin{aligned} \frac{dQ}{dt} &= kA\frac{\Delta T}{\Delta x} \\ \frac{dQ}{dt} &= k\pi DL\frac{T_\mathrm{w} - T_\mathrm{a}}{d} \\ \frac{dQ/dt}{L} &= k\pi D\frac{T_\mathrm{w} - T_\mathrm{a}}{d} \end{aligned}$$

where we have used the surface area of the pipe as $A = \pi D \times L$, the circumference times the length.

2.3 The rate that heat is absorbed by the water is

$$\frac{dQ}{dt} = IA$$

where I is the intensity and A is the surface area of the water. This heat goes into the latent heat of vaporization, $Q = \ell_v m$ so

$$\frac{dQ}{dt} = \ell_v \frac{dm}{dt}$$

and so

$$IA = \ell_v \frac{dm}{dt}$$

If and amount of water dm is evaporated then a volume $dm = \rho\, dV$ is evaporated but the change in volume is $dV = A\ dx$ where dx is the depth of water evaporated. Thus we have that

$$\frac{dm}{dt} = \rho \frac{dV}{dt} = \rho A \frac{dx}{dt}$$

Combining this with the previous result gives us,

$$IA = \ell_v \frac{dm}{dt} = \ell_v \rho A \frac{dx}{dt}$$

and so

$$\frac{dx}{dt} = \frac{I}{\ell_v \rho} = \frac{1.0 \frac{\mathrm{kJ}}{\mathrm{s \cdot m^2}}}{(2260\mathrm{kJ/kg})(1000\mathrm{kg/m^3})} = 4.42 \times 10^{-7} \frac{\mathrm{m}}{\mathrm{s}} = 1.6 \frac{\mathrm{mm}}{\mathrm{hr}}$$

2.4 Let r be the distance from the sun to the planet.

$$I = \frac{P_{\text{sun}}}{4\pi r^2}$$

$$P_{\text{in}} = \epsilon I \pi R^2 = \epsilon \frac{P_{\text{sun}}}{4\pi r^2} \pi R^2$$

$$P_{\text{out}} = \sigma \epsilon 4\pi R^2 T^4$$

$$\sigma \epsilon 4\pi R^2 T^4 = \epsilon \frac{P_{\text{sun}}}{4\pi r^2} \pi R^2$$

$$T^4 = \frac{P_{\text{sun}}}{16\sigma\pi r^2}$$

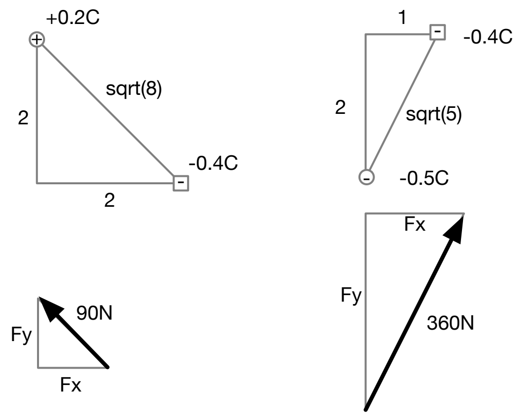
$$\frac{T^4}{T_{\text{E}}^4} = \frac{P_{\text{sun}}}{16\sigma\pi r^2} \frac{16\sigma\pi r_{\text{E}}^2}{P_{\text{sun}}} = \frac{r_{\text{E}}^2}{r^2}$$

$$\frac{T}{T_{\text{E}}} = \sqrt{\frac{r_{\text{E}}}{r}}$$

2.5

$$F_1 = k\frac{|q_1||Q|}{r_1^2} = 9000\text{N}\frac{\text{km}^2}{\text{C}^2}\frac{(0.2\text{C})(0.4\text{C})}{(2\text{km})^2 + (2\text{km})^2} = 90\text{N}$$

$$F_2 = k\frac{|q_2||Q|}{r_2^2} = 9000\text{N}\frac{\text{km}^2}{\text{C}^2}\frac{(0.5\text{C})(0.4\text{C})}{(1\text{km})^2 + (2\text{km})^2} = 360\text{N}$$



$$\frac{F_{1x}}{90\text{N}} = -\frac{2}{\sqrt{8}} \longrightarrow F_{1x} = -64\text{N}$$

$$\frac{F_{1y}}{90\text{N}} = \frac{2}{\sqrt{8}} \longrightarrow F_{1y} = 64\text{N}$$

and

$$\frac{F_{2x}}{360\text{N}} = \frac{1}{\sqrt{5}} \longrightarrow F_{2x} = 161\text{N}$$

$$\frac{F_{2y}}{360\text{N}} = \frac{2}{\sqrt{5}} \longrightarrow F_{2y} = 322\text{N}$$

Now

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

so

$$F_x = F_{1x} + F_{2x} = -64\text{N} + 161\text{N} = 97\text{N}$$

and

$$F_y = F_{1y} + F_{2y} = 64\text{N} + 322\text{N} = 386\text{N}$$

so

$$F = \sqrt{F_x^2 + F_y^2} = 398\text{N}$$

and

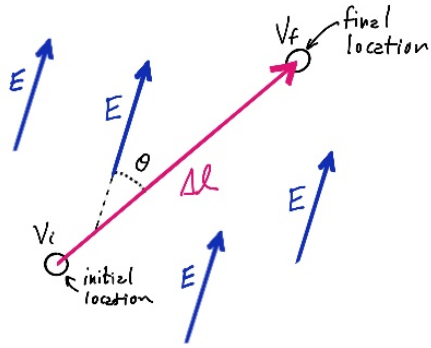
$$\tan\theta = \frac{F_y}{F_x} \longrightarrow \theta = 75.9^\circ$$

2.6

(a) The fundamental relationship between the electric field and electric potential is

$$\Delta V = -\vec{E}\cdot\vec{\Delta\ell} = -E\Delta\ell\cos\theta.$$

It relates the potential difference ΔV between two locations to the vector pointing from the initial position to the final position $\vec{\Delta\ell}$ and the electric field $\vec{E}$ in the region between the initial and final positions.



If we take $\vec{\Delta\ell}$ to be parallel to the plates then since this follows the equipotentials we know that $V_f = V_i$ and thus $\Delta V = 0$ and we can say that

$$\Delta V = -\vec{E}\cdot\vec{\Delta\ell} \;\longrightarrow\; 0 = -\vec{E}\cdot\vec{\Delta\ell}$$

This tells us that the vector $\vec{E}$ must be perpendicular to $\vec{\Delta\ell}$. So now let's take $\vec{\Delta\ell}$ to point perpendicular to the equipotentials from the 18 volt plate to the 12 volt plate then either $\theta = 0$ or $\theta = 180°$. Also the length of $\vec{\Delta\ell}$ is $\Delta\ell = 2\text{mm}$, and $\Delta V = V_f - V_i = 12\text{V} - 18\text{V} = -6\text{V}$. So

$$\Delta V = -E\Delta\ell\cos\theta \;\longrightarrow\; -6\text{V} = -E(2\text{mm})\cos\theta$$

or

$$\frac{6\text{V}}{2\text{mm}} = E\cos\theta$$

so we see that it must be that $\theta = 0$ and the $\vec{E}$ is in the same direction as $\vec{\Delta\ell}$ pointing from the high to low potential plates and with a magnitude.

(b)

$$E = \frac{6\text{V}}{2\text{mm}}$$

(c) Since it is a negatively charged particle it will move against the field from the center towards the higher potential plate. We assume that the energy is conserved so that

$$\Delta K + \Delta U = 0$$

$$K_f - K_i + q\Delta V = 0$$

$$\tfrac{1}{2}mv_f^2 - 0 + q(18\text{V} - 15\text{V}) = 0$$

$$v_f = \sqrt{\frac{-2q(18\text{V}-15\text{V})}{m}} = 253\,\tfrac{\text{m}}{\text{s}}$$

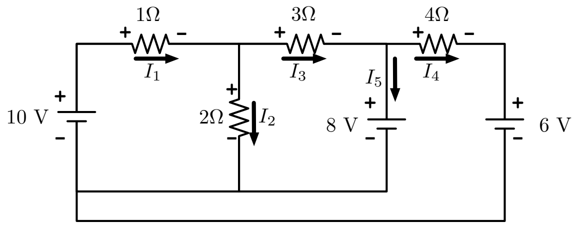
(d) $Q = CV = (5.0\mu\text{F})(6\text{V}) = 30\mu\text{C}$

(e) $RC = (2.0\text{k}\Omega)(5.0\mu\text{F}) = 10\text{ms}$ and $V_S = 0$ so

$$V_C(t) - V_S = (V_C(0) - V_S)e^{-t/RC} \;\longrightarrow\; V_C(t) = (6\text{V})e^{-t/(10\text{ms})}$$

2.7 $P = IV = (3.0\text{A})(12\text{V}) = 36\text{W}.$

2.8 First we label the diagram with directions for the currents and add the associated directions for the potential differences.



$$I_1 - I_2 - I_3 = 0$$

$$I_3 - I_4 - I_5 = 0$$

$$10\text{V} - (1\Omega)I_1 - (2\Omega)I_2 = 0$$

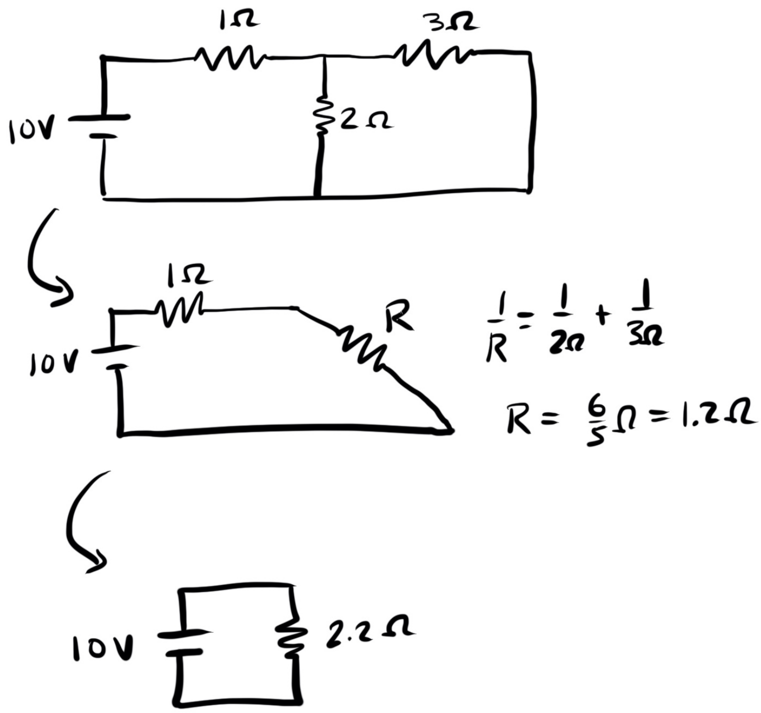
$$(2\Omega)I_2 - (3\Omega)I_3 - 8\text{V} = 0$$

$$8V - (4\Omega)I_4 - 6\text{V} = 0$$

One could now solve for the five unknown currents.

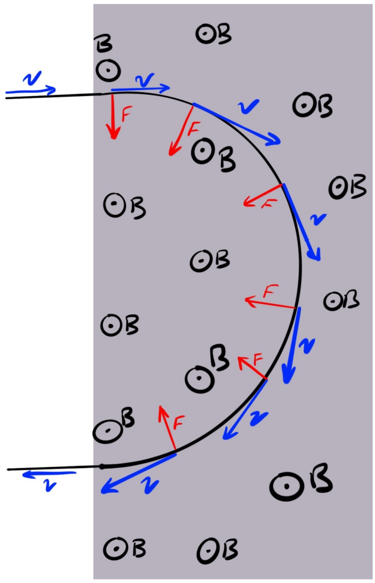
2.9 The 2 and 3 ohm resistors are in parallel and can be thought of as a single resistor with a resistance $\frac{1}{R} = \frac{1}{2\Omega} + \frac{1}{3\Omega} \longrightarrow R = 1.2\Omega$. But this resistor would be in series with the 1 ohm resistor so all three resistors can be combined to a single resistor of 2.2 ohms. So the current drawn will be

$$I = \frac{V}{R} = \frac{10\text{V}}{2.2\Omega} = 4.5\text{A}$$



$$V = IR \rightarrow I = \frac{V}{R} = \frac{10}{2.2}A = 4.5A$$

2.10 Consider the following figure.



By the right-arm-rule the force will be south to begin with, but as the particle curves down the force changes direction so that the force is always toward the center of a circle, and the particle will follow the path of a circle while it is within the magnetic field. We can find the radius of the circle by applying Newton's second law.

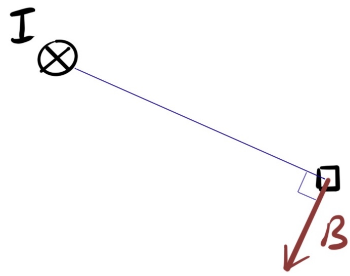
$$\vec{F} = q\vec{v} \times \vec{B} \;\longrightarrow\; F = qvB$$

But also

$$F = ma \;\longrightarrow\; qvB = m\frac{v^2}{r} \;\longrightarrow\; r = \frac{mv}{qB}$$

so the radius of the circle is $\frac{mv}{qB}$.

2.11



2.12 The flux is into the hoop, and the flux is decreasing since the magnet is going away after passing through the hoop. Thus the induced field must be in the same direction as the existing field, into the hoop, away from you. A clockwise current would produce this field.

2.13 The flux will be $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \frac{dB}{dt} A\cos\theta = \left(0.5\frac{\mathrm{T}}{\mathrm{s}}\right) A\cos\theta = (5.0\mathrm{mV})\cos\theta$$

But also $|\mathcal{E}| = IR = (40\mathrm{mA})(0.1\Omega) = 4.0\mathrm{mV}$ so

$$(5.0\mathrm{mV})\cos\theta = 4.0\mathrm{mV} \;\longrightarrow\; \cos\theta = \frac{4}{5} \;\longrightarrow\; \theta = 37^\circ$$

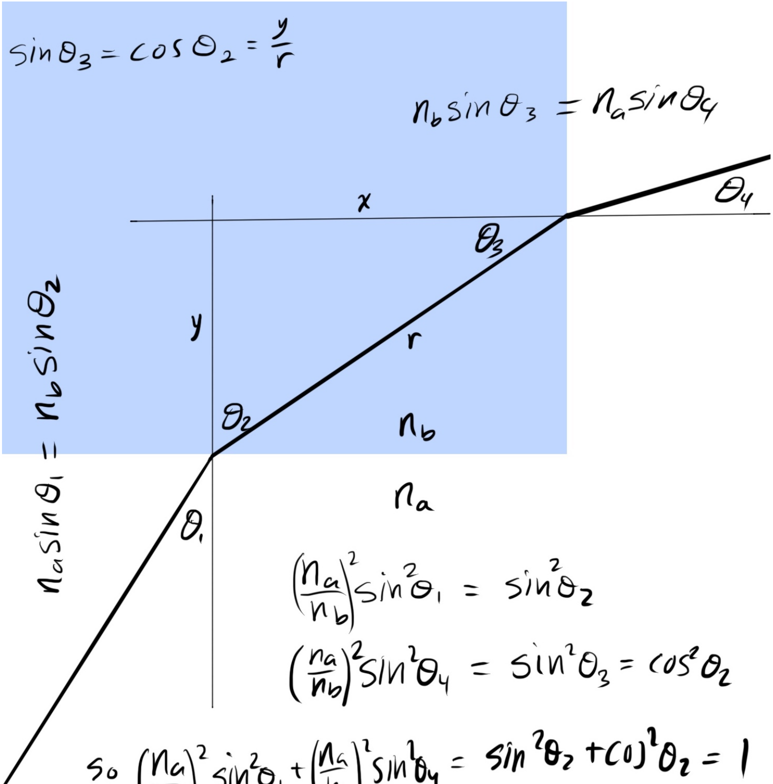
2.14
(a) $u = \frac{1}{2}\epsilon_0 E^2 = 1.1\mu\text{J/m}^3$.
(b) $I = uc = 332\text{W/m}^2$.
(c)

$$c = \frac{\lambda}{T} \quad \longrightarrow \quad T = \frac{\lambda}{c} = \frac{600\text{m}}{3 \times 10^8 \,\frac{\text{m}}{\text{s}}} = 2.0\mu\text{s}$$

(d)

$$\begin{aligned} E(x,t) &= E_0 \cos\left(2\pi\frac{x}{\lambda} - 2\pi\frac{t}{T}\right) \\ &= (500\text{N/C}) \cos\left(2\pi\frac{x}{(600\text{m})} - 2\pi\frac{t}{(2.0\mu\text{s})}\right) \end{aligned}$$

2.15



$$\sin\theta_3 = \cos\theta_2 = \frac{y}{r}$$

$$n_b \sin\theta_3 = n_a \sin\theta_4$$

$$n_a \sin\theta_1 = n_b \sin\theta_2$$

$$\left(\frac{n_a}{n_b}\right)^2 \sin^2\theta_1 = \sin^2\theta_2$$

$$\left(\frac{n_a}{n_b}\right)^2 \sin^2\theta_4 = \sin^2\theta_3 = \cos^2\theta_2$$

so $$\left(\frac{n_a}{n_b}\right)^2 \sin^2\theta_1 + \left(\frac{n_a}{n_b}\right)^2 \sin^2\theta_4 = \sin^2\theta_2 + \cos^2\theta_2 = 1$$

$$\sin^2\theta_1 = \left(\frac{n_b}{n_a}\right)^2 - \sin^2\theta_4 > \left(\frac{n_b}{n_a}\right)^2 - 1$$

So $$\sin\theta_1 > \sqrt{\left(\frac{n_b}{n_a}\right)^2 - 1}$$

$$\theta_1 > \operatorname{asin}\left(\sqrt{(n_b/n_a)^2 - 1}\right) = 61.27^\circ$$

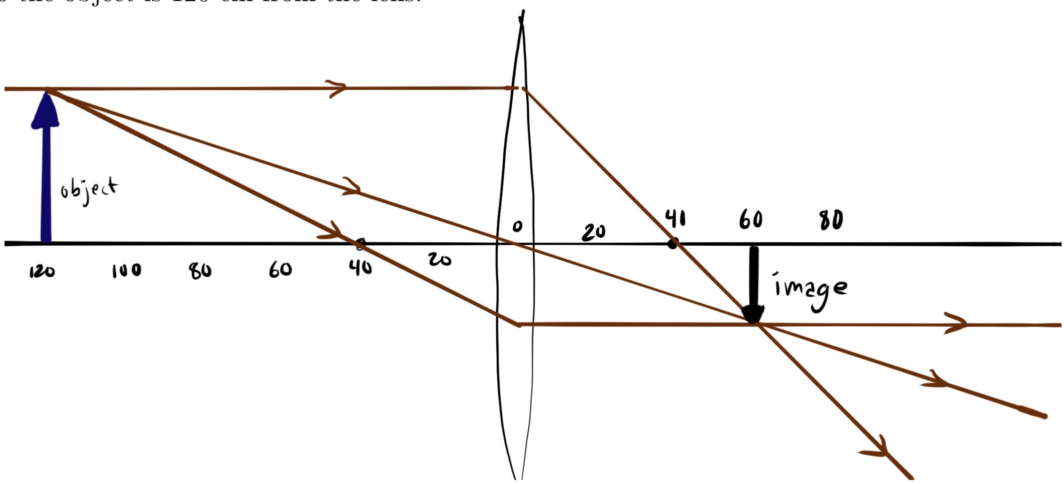
2.16

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{40\text{cm}} - \frac{1}{60\text{cm}} = \frac{3}{120\text{cm}} - \frac{2}{120\text{cm}} = \frac{1}{120\text{cm}}$$

$$\longrightarrow \; d_i = 120\text{cm}$$

So the object is 120 cm from the lens.



2.17

(a)

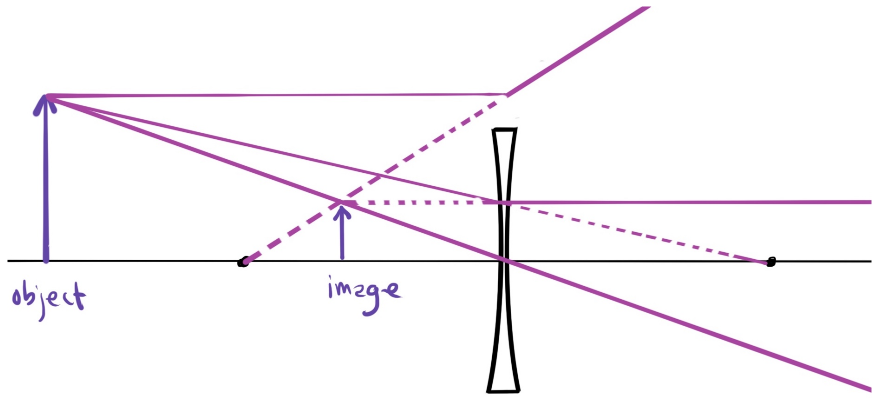
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

so

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-8\text{cm}} - \frac{1}{14\text{cm}} \longrightarrow d_i = -5.09\text{cm}$$

So the image is virtual and on the same side of the lens as the object.

(b)



2.18

(a) $\lambda = \frac{h}{p} = \frac{h}{mv} = 1.00\mu\text{m}$.

(b) $T = \frac{h}{E} = \frac{h}{\frac{1}{2}mv^2} = 2.75\text{ns}$

(c) The phase difference at the first minimum will be $\Delta\phi = \pi$, while the path difference at distance y from the central minimum will be $\Delta x = d\sin\theta$, where $\tan\theta = y/L$

$$\Delta\phi = 2\pi\frac{\Delta x}{\lambda}$$
$$\longrightarrow \pi = 2\pi\frac{d\sin\theta}{\lambda}$$
$$\longrightarrow \sin\theta = \frac{\lambda}{2d} = \frac{1}{1000}$$
$$\longrightarrow \tan\theta = \tan(\sin^{-1}(1/1000)) = 0.001$$
$$\tan\theta = \frac{y}{L} \longrightarrow y = L\tan\theta = 2.00\text{mm}$$

2.19

(a)

$$\begin{aligned}
\Delta K + \Delta U &= 0 \\
\longrightarrow \; K_f - 0 + q\Delta V &= 0 \\
\longrightarrow \; \tfrac{1}{2}mv_f^2 - 0 + q\Delta V &= 0 \\
\longrightarrow \; \tfrac{1}{2}mv_f^2 &= -q\Delta V \\
\longrightarrow \; m^2v_f^2 &= 2m(-q\Delta V) \\
\longrightarrow \; mv_f &= \sqrt{2m(-q\Delta V)} \\
\longrightarrow \; p &= \sqrt{2m(-q\Delta V)} \\
\longrightarrow \; \frac{h}{p} &= \frac{h}{\sqrt{2m(-q\Delta V)}} \\
\longrightarrow \; \lambda &= \frac{h}{\sqrt{2m(-q\Delta V)}} \\
\longrightarrow \; \lambda &= \frac{hc}{\sqrt{2mc^2(-q\Delta V)}} \\
\longrightarrow \; \lambda &= \frac{1240\text{eV}\cdot\text{nm}}{\sqrt{2mc^2(-q\Delta V)}}
\end{aligned}$$

(b) For an electron $mc^2 = 0.511 \times 10^6\text{eV}$ and $q = -e$. So with $\Delta V = 1.5045n^2$ Volts this leads to

$$\sqrt{2mc^2(-q\Delta V)} = \sqrt{2(0.511 \times 10^6\text{eV})(e(1.5045\text{V})n^2)} = (1240\text{eV})n$$

$$\begin{aligned}
\lambda &= \frac{1240\text{eV}\cdot\text{nm}}{\sqrt{2mc^2(-q\Delta V)}} \\
&= \frac{1240\text{eV}\cdot\text{nm}}{(1240\text{eV})n} = \frac{1}{n}\text{nm}
\end{aligned}$$

2.20

(a) In either emission or absorption we have that

$$hf = |E_{n_{\text{initial}}} - E_{n_{\text{final}}}|$$

so in this case we have $E_n = \varepsilon n^2$ so

$$hf = |\varepsilon n_{\text{initial}}^2 - \varepsilon n_{\text{final}}^2| = \varepsilon |n_{\text{initial}}^2 - n_{\text{final}}^2|$$

so

$$\frac{hc}{\lambda} = \varepsilon |n_{\text{initial}}^2 - n_{\text{final}}^2|$$

$$\longrightarrow \lambda = \frac{hc}{\varepsilon |n_{\text{initial}}^2 - n_{\text{final}}^2|} = \frac{620\text{nm}}{|n_{\text{initial}}^2 - n_{\text{final}}^2|}$$

Putting in pairs of numbers selected from $\{0, 1, 2, 3\}$ we get

$$\lambda_{01} = 620\text{nm}$$

$$\lambda_{02} = 155\text{nm}$$

$$\lambda_{03} = 68.9\text{nm}$$

$$\lambda_{12} = 207\text{nm}$$

$$\lambda_{13} = 77.5\text{nm}$$

$$\lambda_{23} = 124\text{nm}$$

(b) The transition from 3 to 0 gives the shortest wavelength photon.
(c) The transition from 1 to 0 gives the longest wavelength photon.

2.21

$$\psi^2 = \psi_A^2 + \psi_B^2 + 2\psi_A\psi_B \cos\phi$$

but $N \propto \psi^2$ so

$$N = N_A + N_B + 2\sqrt{N_A}\sqrt{N_B}\cos\phi$$

(a)

$$\frac{N - N_A - N_B}{2\sqrt{N_A}\sqrt{N_B}} = \cos\phi$$

$$\cos\phi = \frac{90 - 30 - 20}{2\sqrt{30}\sqrt{20}} \longrightarrow \phi = 35.264^\circ = 0.615\text{rad}$$

(b)

$$N = 30 + 20 + 2\sqrt{30}\sqrt{20}\cos\phi$$

The maximum will be achieved when $\cos\phi = 1$ and in this case we have

$$N = 30 + 20 + 2\sqrt{30}\sqrt{20} = 99.0$$

The minimum will be achieved when $\cos\phi = -1$ and in this case we have

$$N = 30 + 20 - 2\sqrt{30}\sqrt{20} = 1.0$$

2.22

$$\Delta x \ \Delta p \geq \frac{\hbar}{2} \ \longrightarrow \ \Delta p \geq \frac{\hbar}{2\Delta x}$$

But $\Delta p = m\Delta v$ so

$$\Delta v = \frac{\Delta p}{m} \geq \frac{\hbar}{2m\Delta x} \approx 232 \, \frac{\mathrm{m}}{\mathrm{s}}$$

2.23 The nucleus has two less neutrons and two less protons.

2.24 We will do this problem twice. First we start with the decay rate equation.

$$R = R_0 \; 2^{-t/T_{1/2}}$$

so

$$\frac{R}{R_0} = 2^{-t/T_{1/2}}$$

and

$$\log\left(\frac{R}{R_0}\right) = \log\left(2^{-t/T_{1/2}}\right) = -\frac{t}{T_{1/2}} \log(2)$$

$$T_{1/2} = -\frac{\log(2)}{\log\left(\frac{R}{R_0}\right)} t$$

At $t = 1.0$ days $R = 1500\frac{1}{\mathrm{s}}$ while $R_0 = 2000\frac{1}{\mathrm{s}}$ so we find

$$T_{1/2} = -\frac{\log(2)}{\log\left(\frac{1500}{2000}\right)} (1.0 \text{ days}) = 2.41 \text{ days}$$

Alternatively we can follow this second method to get to the same result:

$$R = R_0 e^{-\lambda t}$$

so

$$\frac{R}{R_0} = e^{-\lambda t}$$

and

$$\log\left(\frac{R}{R_0}\right) = -\lambda t$$

and

$$\lambda = -\frac{\log\left(\frac{R}{R_0}\right)}{t}$$

At $t = 1.0$ days $R = 1500\frac{1}{\mathrm{s}}$ while $R_0 = 2000\frac{1}{\mathrm{s}}$ so we find

$$\lambda = -\frac{\log(1500/2000)}{(1.0 \text{ days})} = 0.28768 \frac{1}{\text{days}}$$

Now we know that at $t = T_{1/2}$ that $R = \frac{1}{2} R_0$ so that

$$\tfrac{1}{2} R_0 = R_0 e^{-\lambda T_{1/2}}$$

so

$$\tfrac{1}{2} = e^{-\lambda T_{1/2}}$$

so

$$\log(1/2) = -\lambda T_{1/2}$$

so

$$T_{1/2} = -\frac{\log(1/2)}{\lambda} = 2.41 \text{ days}$$