Exam 2 Solns

1/

$$\cancel{K_i}^{\,0 \text{ at rest}} + U_i = K_f + U_f$$

$$\cancel{\frac{kq_1q_2}{r}} = \frac{1}{2}mv^2 + \cancel{\frac{kq_1q_2}{r}} + \frac{kq_2q_3}{r_{23}} + \frac{kq_3q_1}{r_{31}}$$

same term

$$\frac{1}{2}mv^2 = -kq_3\left(\frac{q_2}{r_{23}} + \frac{q_1}{r_{31}}\right)$$

$$v = \left[\frac{-2kq_3}{m}\left(\frac{q_2}{r_{23}} + \frac{q_1}{r_{31}}\right)\right]^{1/2}$$

$$v = \left[\frac{-2(9\cdot10^9)(1.6\cdot10^{-19})}{1.67\cdot10^{-27}}\left(\frac{-6\cdot10^{-9}}{3} + \frac{2\cdot10^{-9}}{5}\right)\right]^{1/2} \text{ in } \frac{m}{s}$$

2/

$$V(2,2) = A(2\cdot2^2 - 2\cdot2^2) = 0\,V$$

$$\vec{E}(x,y) = -\frac{\partial V}{\partial x}\hat{x} - \frac{\partial V}{\partial y}\hat{y}$$

where $\frac{\partial V}{\partial x} = Ay^2 - Ay(2x)$

$\frac{\partial V}{\partial y} = Ax(2y) - Ax^2$

$$= -A\left[(y^2-2xy)\hat{x} + (2xy-x^2)\hat{y}\right]$$

$$\vec{E}(2,2) = -A\left[(2^2-2^3)\hat{x} + (2^3-2^2)\hat{y}\right]$$

$$= -A\left[-4\hat{x} + 4\hat{y}\right]$$

$$\vec{E} = 4A\hat{x} - 4A\hat{y}$$

3/ $\vec{E} = 0$ and $V = 150\,V$

$\vec{E} = 0$ inside a conductor

The entire conductor is an equipotential surface/volume. $\Delta V = \int \vec{E}\cdot d\vec{r} = 0$

so $V = 150$ at edge, 150 V everywhere.

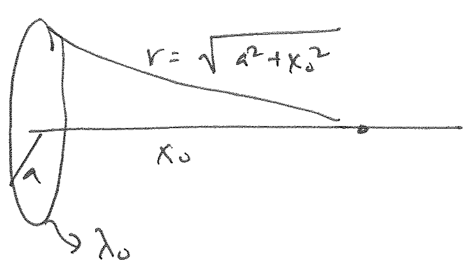
4/ $C = \frac{Q}{V}$ $\quad C = \frac{50\cdot 10^{-9}}{70} = \frac{5}{7}\,nC$

$U = \frac{1}{2} C V^2$

$\sqrt{\frac{2U}{C}} = V$

$\sqrt{\frac{2(100)}{\frac{5}{7}\cdot 10^{-9}}} = V$ in Volts

5/



$\Delta V = \frac{k\,\Delta q}{r}$ $\quad$ here, $\Delta q = \lambda\,\Delta s$ along arc

$r = \sqrt{a^2 + x_0^2}$

$$V = \int \frac{-\lambda_0 k\, ds}{\sqrt{a^2+x_0^2}}$$

$$V = \frac{-\lambda_0 k}{\sqrt{a^2+x_0^2}} \int ds$$

$$V = \frac{-\lambda_0 k\,(2\pi a)}{\sqrt{a^2+x_0^2}}$$

potential is negative because the charge is negative

6/

$$\Delta V = -\int \vec{E}\cdot d\vec{l}$$

$$= -\vec{E}\cdot\Delta\vec{r}$$ if $\vec{E}$ is uniform (it is!)

$$= -(13)(3)$$

generally, $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

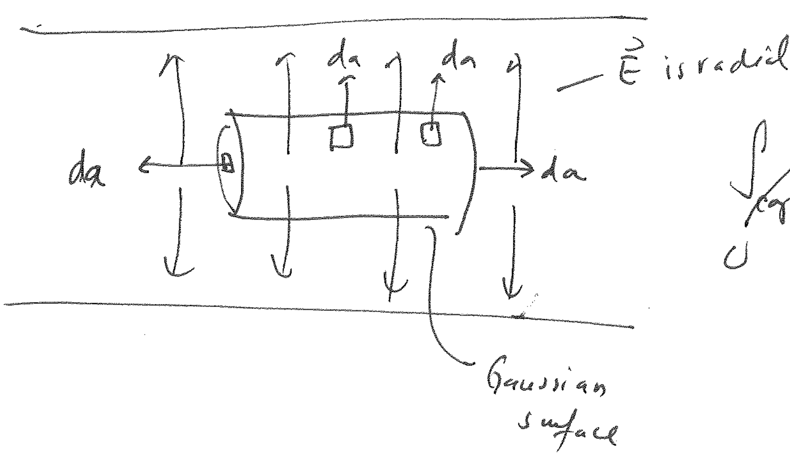
here, $\vec{E} = 13\,\frac{N}{C}\,\hat{y}$, $\Delta\vec{r} = -7\hat{x} + 3\hat{y}$

$$\Delta V = -39\,V$$

so $\Delta V = V_B - V_A$

$$V_B = V_A + \Delta V = 11 - 39 = -28\,V$$

7/



$$\oint \vec{E}\cdot d\vec{a} = q_{enc}/\epsilon_0$$

$$\underbrace{\cancel{\int_{cap}}}_{0} + \int_{side} + \underbrace{\cancel{\int_{cap}}}_{0} = \frac{\rho}{\epsilon_0}\cdot \text{Vol of small cylinder}$$

$$\int_{side} \vec{E}\cdot da = \frac{\rho}{\epsilon_0}\pi r^2 l$$

$$E(2\pi r l) = \frac{\rho}{\epsilon_0}\pi r^2 l$$

$$E = \frac{\rho r}{2\epsilon_0}$$

8/ (a) A & B. Using $J = \sigma E$,.. A+B has a higher J than C+D since area is less.

σ is same for A+B, C+D

therefore, E higher for A+B.

(b) U = energy density × volume

$= \frac{1}{2}\epsilon_0 E^2 \cdot \text{Volume}$

$= \frac{1}{2}(8.9\cdot10^{-12})(3\cdot10^{6})(1\cdot10^{-2})^3$ in Joules

9/ $C \equiv \frac{Q}{V}$ for a parallel plate cap,

$$\underbrace{V = Ed \quad \text{and} \quad E = \frac{\eta}{\epsilon_0}}$$

$V = \frac{\eta d}{\epsilon_0}$ where $\eta = \frac{Q}{A}$

$V = \frac{Q \cdot d}{A\epsilon_0}$

now using the defin $C \equiv \frac{Q}{V} = \frac{Q}{Qd/A\epsilon_0}$

$$C = \frac{\epsilon_0 A}{d}$$

10/ (a) $d = v_D t$ here, $t = 0.5\,s$

and $I = nqv_d A$

so $v_d = \frac{I}{nqA}$

$d = \frac{I \cdot t}{nqA} = \frac{300\,(0.5)}{(8.5\cdot 10^{28})(1.6\cdot 10^{-19})(0.21\text{ cm}^2)\left(\frac{1\text{ m}}{10^{-2}}\right)^2}$ in m

(b) $I = \frac{\Delta q}{\Delta t}$

$I\Delta t = \Delta q$

$(300)(0.5) = 150\,C$