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You are climbing a vertical rock face along the shore of a lake, and want to make a phone call. Unfortunately you have zero bars of reception, because of the interference between the direct signal cellphone tower and the signal reflected from the lake. You calculate that the direct and reflected signal paths are

$$r_{\text{direct}} = L_0 - \frac{y}{4\alpha}$$
$$r_{\text{reflected}} = L_0 + \frac{y}{4\alpha}$$

where y is your height above the water, $\alpha = 10$ and L_0 is a constant. The wavelength of the signal is 33 cm. How far do you need to move up the face in order to get the best reception.



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Solution:

The path difference is

$$\Delta r = (L_0 + \frac{y}{4\alpha}) - (L_0 - \frac{y}{4\alpha}) = \frac{y}{2\alpha}$$

So the phase difference due to the path is

$$\Delta \phi_{\text{path}} = 2\pi \frac{\Delta r}{\lambda} = \pi \frac{y}{\alpha \lambda}$$

so The reflection from the water causes an inversion, so $\Delta \phi_{\text{reflection}} = \pi$ and the total phase difference between the paths of

$$\Delta \phi = \Delta \phi_{\text{path}} + \Delta \phi_{\text{reflection}} = \pi \frac{y}{\alpha \lambda} + \pi$$

At the initial position there is a minimum of the signal so it must be that $\Delta \phi = m\pi$ with m an odd integer.

$$m\pi = \pi \frac{y_i}{\alpha \lambda} + \pi$$
 with m odd

As I climb higher the phase difference increases at some point it will increase by π to $\Delta \phi = m\pi + \pi$ and give a maximum signal.

$$m\pi + \pi = \pi \frac{y_f}{\alpha \lambda} + \pi$$

Subtracting these two equations we find

$$[m\pi + \pi] - [m\pi] = \left[\pi \frac{y_f}{\alpha \lambda} + \pi\right] - \left[\pi \frac{y_i}{\alpha \lambda} + \pi\right]$$
$$\pi = \pi \frac{y_f - y_i}{\alpha \lambda}$$
$$\longrightarrow \Delta y = y_f - y_i = \alpha \lambda = 330 \text{cm}$$