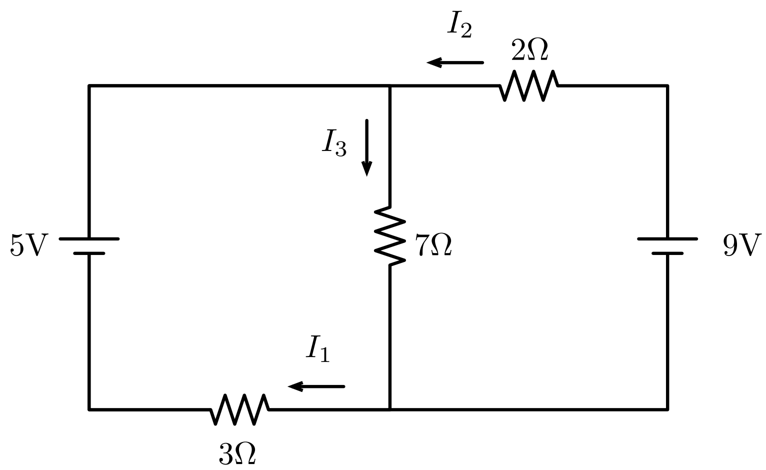
Find the three currents I_1, I_2 and I_3



Physics 11 - Quiz 11 3/16/2020

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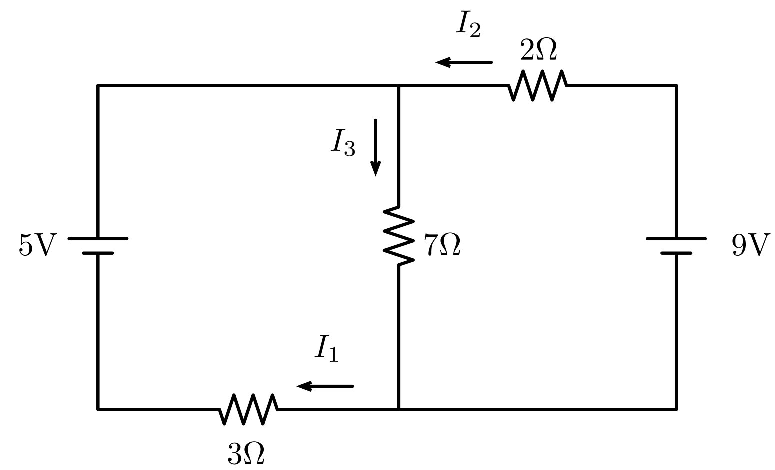
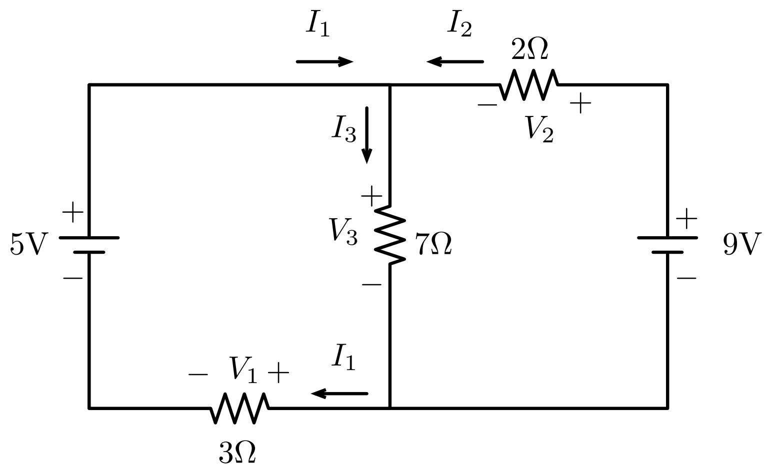


Solution: First we add the plus and minus signs to the diagram.



Now we write out two Kirckhhoff's loop rules and one junction rule.

$$(5\text{V}) - V_3 - V_1 = 0$$
$$V_3 + V_2 - (9\text{V}) = 0$$
$$I_1 + I_2 - I_3 = 0$$

We now use Ohm's Law three times.

$$V_1 = (3\Omega)I_1$$
$$V_2 = (2\Omega)I_2$$
$$V_3 = (7\Omega)I_3$$

Putting these three equations into the original three equations we find

$$(5\text{V}) - (7\Omega)I_3 - (3\Omega)I_1 = 0$$
$$(7\Omega)I_3 + (2\Omega)I_2 - (9\text{V}) = 0$$
$$I_1 + I_2 - I_3 = 0$$

Dividing the first two equations by 1Ω we get the following.

$$(5\text{A}) - 7I_3 - 3I_1 = 0$$
$$7I_3 + 2I_2 - (9\text{A}) = 0$$
$$I_1 + I_2 - I_3 = 0$$

This is three equations in the three unknown currents. Solving the last of these equations for I_3 we have

$$I_3 = I_1 + I_2.$$

We can put this into the first two equations to eliminated I_3.

$$(5\text{A}) - 7(I_1 + I_2) - 3I_1 = 0$$
$$7(I_1 + I_2) + 2I_2 - (9\text{A}) = 0$$

This gives us two equations in two unknowns. We can simplify them a little to get the following.

$$(5\text{A}) - 10I_1 - 7I_2 = 0$$
$$7I_1 + 9I_2 - (9\text{A}) = 0$$

This is two equations in the two unknowns. Solving the last of these equations for I_2 we have

$$I_2 = (1\text{A}) - \frac{7}{9}I_1.$$

Putting this into the first equation we find

$$(5\text{A}) - 10I_1 - 7\left((1\text{A}) - \frac{7}{9}I_1\right) = 0$$
$$(45\text{A}) - 90I_1 - 7\left((9\text{A}) - 7I_1\right) = 0$$
$$\longrightarrow\ (45\text{A} - 63\text{A}) + (49 - 90)I_1 = 0$$
$$\longrightarrow\ -(18\text{A}) - 41I_1 = 0$$
$$\longrightarrow\ I_1 = -\frac{18}{41}\text{A}$$

We can put this back into our expression for I_2 to find

$$I_2 = (1\text{A}) - \frac{7}{9}I_1 = (1\text{A}) - \frac{7}{9}(-\frac{18}{41}\text{A}) = \frac{55}{41}\text{A}.$$

We can put this into our expression for I_3 to find

$$I_3 = I_1 + I_2 = -\frac{18}{41}\text{A} + \frac{55}{41}\text{A} = \frac{37}{41}\text{A}$$